1.] A] Solve $(1+e^{x/y})dx + e^{x/y}(1-\frac{x}{y})dy = 0$

Given D.E is $(1+e^{x/y})dx + e^{x/y}(1-\frac{x}{y})dy = 0$ → ①

compare eqⁿ ① with $Mdx + Ndy = 0$

here $M = 1+e^{x/y}$, $N = (1-\frac{x}{y})e^{x/y}$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(1+e^{x/y})$$

$$= 0 + e^{x/y}\cdot\frac{\partial}{\partial y}\left(\frac{x}{y}\right)$$

$$= e^{x/y}\,x\cdot\frac{\partial}{\partial y}\left(\frac{1}{y}\right)$$

$$= e^{x/y}\,x\left(\frac{-1}{y^2}\right)$$

$$= \frac{-x}{y^2}e^{x/y}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(e^{x/y}\left(1-\frac{x}{y}\right)\right)$$

$$= e^{x/y}\left(0-\frac{1}{y}(1)\right) + \left(1-\frac{x}{y}\right)e^{x/y}\frac{\partial}{\partial x}\left(\frac{x}{y}\right)$$

$$= e^{x/y}\left(\frac{-1}{y}\right) + \left(1-\frac{x}{y}\right)e^{x/y}\left(\frac{1}{y}(1)\right)$$

$$= \cancel{e^{x/y}\left(\frac{-1}{y}\right)} + \cancel{\frac{1}{y}e^{x/y}} - \frac{x}{y^2}e^{x/y}$$

$$= \frac{-x}{y^2}e^{x/y}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eqⁿ ① is an exact diff eqⁿ

The General Solution of eqⁿ ① is given by

$$\int\limits_{y \text{ is constant}} M\,dx + \int (\text{Terms of } N \text{ free from } x)\,dy = c$$

$$\int\limits_{y \text{ constant}} (1+e^{x/y})\,dx + \int (0)\,dy = c$$

$$x + \frac{e^{x/y}}{1/y} + c_1 = c$$

$$x + y\cdot e^{x/y} = c - c_1 = d, \text{ where } d = c - c_1$$

$$x + y\cdot e^{x/y} = d$$

B] Solve $y^2dx + (x^2 - xy - y^2)dy = 0$

Given de is $y^2dx + (x^2 - xy - y^2)dy = 0$ → ①

compare eqⁿ ① with $Mdx + Ndy = 0$

Here $M = y^2$ & $N = x^2 - xy - y^2$

$$\frac{\delta M}{\delta y} = \frac{\delta}{\delta y}(y^2) = 2y \qquad \frac{\delta N}{\delta x} = \frac{\delta}{\delta x}(x^2 - xy - y^2) = 2x - y$$

$$\therefore \frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$$

eq^n ① is not an exact diff eq^n

But M & N are homogenous functions of the same degree

eq^n ① is a homogenous diff eq^n

$$\text{Now, } Mx + Ny = y^2(x) + (x^2 - xy - y^2)y$$
$$= \cancel{y^2x} + yx^2 - \cancel{y^2x} - y^3$$
$$= x^2y - y^3 \neq 0$$

$$I.F = \frac{1}{Mx+Ny}$$

$$= \frac{1}{x^2y - y^3}$$ multiplying on b.s of eq^n ① by I.F $= \frac{1}{x^2y - y^3}$

$$\frac{1}{x^2y - y^3}\left[y^2dx + (x^2 - xy - y^2)dy\right] = 0 \times \frac{1}{x^2y - y^3}$$

$$\frac{y^2}{x^2y - y^3}dx + \frac{x^2 - xy - y^2}{x^2y - y^3}dy = 0$$

$$\frac{y^2}{x^2y - y^3}dx + \frac{x^2 - xy - y^2}{y(x^2 - y^2)}dy = 0 \quad \text{—②}$$

compare eq^n ② with $M_1dx + N_1dy = 0$

$$\text{here } M_1 = \frac{y}{x^2 - y^2} \quad \& \quad N_1 = \frac{x^2 - xy - y^2}{y(x^2 - y^2)} = \frac{(x^2 - y^2) - xy}{(x^2 - y^2)y}$$

$$\frac{\delta M_1}{\delta y} = \frac{\delta}{\delta y}\left(\frac{y}{x^2 - y^2}\right) \qquad \frac{\delta N_1}{\delta x} = \frac{\delta}{\delta x}\left(\frac{1}{y} - \frac{x}{x^2 - y^2}\right)$$

$$= \frac{(x^2 - y^2)(1) - y(0 - 2y)}{(x^2 - y^2)^2} \qquad = 0 - \left[\frac{(x^2 - y^2)(1) - x(2x - 0)}{(x^2 - y^2)^2}\right]$$

$$\frac{\delta M_1}{\delta y} = \frac{x^2+y^2}{(x^2-y^2)^2} \qquad \frac{\delta N_1}{\delta x} = -\left[\frac{-y^2-x^2}{(x^2-y^2)^2}\right] = \frac{x^2+y^2}{(x^2-y^2)^2}$$

$$\therefore \frac{\delta M_1}{\delta y} = \frac{\delta N_1}{\delta x}$$

eqⁿ ② is an exact diff eqⁿ

The G.S of eqⁿ ② is given by

$$\underset{y\text{ constant}}{\int M_1\,dx} + \int (\text{The terms of } N_1 \text{ free from } x)\,dy = c$$

$$\underset{y\text{ as constant}}{\int \frac{y}{x^2-y^2}\,dx} + \int \frac{1}{y}\,dy = c$$

$$y\int \frac{1}{x^2-y^2}\,dx + \log y = \log c$$

$$\cancel{y}\left[\frac{1}{2\cancel{y}}\cdot\log\left(\frac{x-y}{x+y}\right)\right] + \log y = \log c$$

lo

$$\log\left(\frac{x-y}{x+y}\right)^{1/2} + \log y = \log c$$

$$\log\left(\left(\frac{x-y}{x+y}\right)^{1/2}\cdot y\right) = \log c$$

$$y\cdot\left(\frac{x-y}{x+y}\right)^{1/2} = c$$

which is G.S of eqⁿ ② is same as G.S of eqⁿ ①

2] A] Solve $(D^2-2D)y = e^x \sin x$, Method of variation of parameters

G.T $(D^2-2D)y = e^x \sin x \rightarrow$ ①

Compare above eqⁿ with Standard form

here $P=-2$, $Q=0$ & $R = e^x \sin x$

$f(D) = D^2-2D$

A.E is $f(m)=0$

$m^2-2m=0$

$m(m-2)=0$

$m = 2, 0$ Roots are real & distinct

$$C.F(y_c) = c_1 e^{0x} + c_2 e^{2x}$$

$$y_c = c_1 + c_2 e^{2x}$$

$$= c_1 y_1 + c_2 y_2$$

here $y_1 = 1$, $y_2 = e^{2x}$

$$w(y_1, y_2) = y_1 y_2' - y_2 y_1'$$

$$= 1(2 \cdot e^{2x}) - e^{2x}(0)$$

$$W = 2 \cdot e^{2x}$$

let $y_p = v_1 y_1 + v_2 y_2$ be the P.I of eq^n 1

$$v_1 = -\int \frac{R y_2}{W} dx$$

$$= -\int \frac{e^x \sin x \cdot \cancel{e^{2x}}}{2 \cdot \cancel{e^{2x}}} dx$$

$$= -\frac{1}{2} \int e^x \sin x \, dx$$

$$\left[\because \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx) \right]$$

$a=1, b=1$

$$= \frac{-1}{2} \left[\frac{e^x}{1^2+1^2} (1 \sin 1x - 1 \cos 1x) \right]$$

$$= \frac{-1}{2} \left[\frac{e^x}{2} (\sin x - \cos x) \right]$$

$$v_1 = -\frac{e^x}{4} (\sin x - \cos x)$$

$$v_2 = +\int \frac{R y_1}{W} dx$$

$$= \int \frac{e^x \sin x \cdot 1}{2 \cdot e^{2x}} dx$$

$$= \frac{1}{2} \int e^x e^{-2x} \sin x \, dx$$

$$= \frac{1}{2} \int e^{-x} \sin x \, dx$$

$(a=-1, b=1)$

$$= \frac{1}{2} \left[\frac{e^{-1x}}{(-1)^2+1^2} (-1 \sin 1x - \cos 1x) \right]$$

$$= \frac{1}{2} \left[\frac{e^{-x}}{2} (-\sin x - \cos x) \right]$$

$$v_2 = \frac{-e^{-x}}{2} (\sin x + \cos x)$$

$$y_p = v_1 y_1 + v_2 y_2$$

$$= \left[-\frac{e^x}{4} (\sin x - \cos x) \right](1) + \left[-\frac{e^{-x}}{4} (\sin x + \cos x) \right] e^{2x}$$

$$= \frac{-e^{+x}}{4} (\sin x - \cos x) - \frac{e^x}{4} (\sin x + \cos x)$$

$$= \frac{-2e^{x}}{4}\sin x + \cancel{\frac{e^{x}}{4}\cos x} - \cancel{\frac{e^{x}}{4}\cos x}$$

$$= \frac{-2e^{x}}{4}\sin x = \frac{-1}{2}e^{x}\sin x$$

The G.S of eqn ① is $y = y_c + y_p$

$y = c_1 + c_2 e^{2x} - \frac{1}{2}e^{x}\sin x$, where c_1, c_2 are orbitary Constants

2] B] Solve $(D^3 - 7D^2 + 14D - 8)y = e^{x} + \sin^3 x + 2$

G.T $(D^3 - 7D^2 + 14D - 8)y = e^{x} + \sin^3 x + 2 \rightarrow$ ①

Compare above eqn with $f(D)y = Q(x)$

A.E $f(m) = 0$

$m^3 - 7m^2 + 14m - 8 = 0$

$(m-2)(m^2 - 5m + 4) = 0$

$m = 2, 4, 1$

$$C.F = c_1 e^{4x} + c_2 e^{2x} + c_3 e^{1x}$$

$$PI = \frac{1}{f(D)} \cdot Q(x) = \frac{1}{(D^3 - 7D^2 + 14D - 8)} \cdot (e^{x} + \sin^3 x + 2)$$

$$P.I = \frac{1}{D^3 - 7D^2 + 14D - 8} e^{x} + \frac{1}{D^3 - 7D^2 + 14D - 8} \sin^3 x + \frac{1}{D^3 - 7D^2 + 14D - 8} 2$$

consider $y_p = y_{p_1} + y_{p_2} + y_{p_3}$

$$y_{p_1} = \frac{1}{D^3 - 7D^2 + 14D - 8} e^{x}$$

Sub D by '1' DN = 0

$$= \frac{1}{(D-2)(D-1)(D-4)} e^{x} = \frac{1}{(D-2)(D-4)} \cdot \left(\frac{1}{D-1} e^{x}\right)$$

$$= \frac{1}{(1-2)(1-4)} \left(\frac{1}{D-1} e^{x}\right)$$

$$\left(\because \frac{1}{(D-a)^{k}} e^{ax} = \frac{x^{k}}{k!} e^{ax}\right)$$

$$= \frac{1}{(-1)(-3)} \cdot \frac{x^{1}}{1!} e^{x}$$

$$y_{P_1} \Rightarrow \frac{x}{3} e^{x}$$

$$\left[\because \sin 3x = 3\sin x - 4\sin^{3} x\right]$$

$$y_{P_2} = \frac{1}{D^3 - 7D^2 + 14D - 8} \cdot \sin^{3} x$$

$$\left[\therefore \sin^{3} x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x\right]$$

$$= \frac{3}{4} \cdot \frac{1}{D^3 - 7D^2 + 14D - 8} \sin x - \frac{1}{4} \cdot \frac{1}{D^3 - 7D^2 + 14D - 8} \sin 3x$$

Replace D^2 by -1^2 $DN \neq 0$ Replace D^2 by -3^2 $DN \neq 0$

$$= \frac{3}{4} \frac{1}{D(-1^2) - 7(-1^2) + 14D - 8} \sin x - \frac{1}{4}\left(\frac{1}{D(-3^2) - 7(-3^2) + 14D - 8} \sin 3x\right)$$

$$= \frac{3}{4} \frac{1}{-D + 7 + 14D - 8} \sin x - \frac{1}{4}\left(\frac{1}{-9D + 63 + 14D - 8} \sin 3x\right)$$

$$= \frac{3}{4} \cdot \frac{1}{13D - 1} \sin x - \frac{1}{4} \cdot \frac{1}{5D + 55} \sin 3x$$

$$= \frac{3}{4} \cdot \frac{1}{13D - 1} \times \frac{13D + 1}{13D + 1} \sin x - \frac{1}{4} \frac{1}{5D + 55} \times \frac{5D - 55}{5D - 55} \sin 3x$$

$$= \frac{3}{4} \frac{13D + 1}{169D^2 - 1} \sin x - \frac{1}{4} \frac{5D - 55}{25D^2 - 3025} \sin 3x$$

Replace D^2 by -1^2 $DN \neq 0$ Replace D^2 by -3^2 $DN \neq 0$

$$= \frac{3}{4} \frac{13D + 1}{-169 - 1} \sin x - \frac{1}{4} \frac{5D - 55}{-225 - 3025} \sin 3x$$

$$= \frac{3}{4} \frac{13D + 1}{-170} \sin x - \frac{1}{4} \frac{5D - 55}{-3250} \sin 3x$$

$$= \frac{-3}{4\times170}(13D+1)\sin x + \frac{1}{4\times3250}(5D-55)\sin 3x$$

$$= \frac{-3}{680}\left(13\frac{d}{dx}\sin x + \sin x\right) + \frac{1}{13000}\left(5\frac{d}{dx}\sin 3x - 55\sin 3x\right)$$

$$= \frac{-3}{680}(13\cos x + \sin x) + \frac{1}{13000}(5\times 3\,\cos 3x - 55\sin 3x)$$

$$y_{P_2} = \frac{-3}{680}(13\cos x + \sin x) + \frac{1}{13000}(15\cos 3x - 55\sin 3x)$$

$$y_{P_3} = \frac{1}{D^3 - 7D^2 + 14D - 8}\cdot 2$$

$$= \frac{1}{D^3 - 7D^2 + 14D - 8}\, 2\cdot e^{0x}$$

Sub D by '0' $DN \neq 0$

$$= \frac{1}{(0)^3 - 7(0)^2 + 14(0) - 8}\, 2\cdot e^{0x} = -\frac{2}{8}e^{0x} = -\frac{2}{8}$$

$$y_{P_3} = -\frac{1}{4}$$

$$y_P = y_{P_1} + y_{P_2} + y_{P_3}$$

$$y_P = \frac{x}{3}e^{x} - \frac{3}{680}(13\cos x + \sin x) + \frac{1}{13000}(15\cos 3x - 55\sin 3x) - \frac{1}{4}$$

The G.S of eq^n ① is $y = y_c + y_p$

$$y = c_1 e^{4x} + c_2 e^{2x} + c_3 e^{1x} + \frac{x}{3}e^{x} - \frac{3}{680}(13\cos x + \sin x) + \frac{1}{13000}(15\cos 3x - 55\sin 3x) - \frac{1}{4}$$

3] A] A condenser of capacity c discharged through an inductance L & resistance R in series and the charge q at time t satisfies the eq^n $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$. G.T $L = 0.25H$, $R = 250$ ohms, $c = 2\times10^{-6} f$, & that when $t=0$, charge q is 0.002 coloumbs & current $\frac{dq}{dt} = 0$. obtain the value of q in terms of t

G.T $L = 0.25H$, $R = 250\Omega$, $C = 2\times10^{-6}F$

Given d.e is $L \cdot \frac{d^2q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{q}{c} = 0$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

$$\frac{d^2q}{dt^2} + \frac{250}{0.25}\frac{dq}{dt} + \frac{q}{0.25\times2\times10^{-6}} = 0, \quad \frac{d}{dt} = D$$

The operator form of above eq^n is

$$D^2q + 1000Dq + (2\times10^6)q = 0$$

$$(D^2 + 1000D + (2\times10^6))q = 0 \rightarrow (1)$$

compare eq^n (1) with $f(D) = Q(t)$

here $f(D) = D^2 + 1000D + 2\times10^6$ & $Q(t) = 0$

A.E is $f(m) = 0$

$m^2 + 1000m + 2\times10^6$ — here $a=1$, $b=1000$, $c=2\times10^6$

$$m = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1000\pm\sqrt{(10^3)^2 - 8\times10^6}}{2(1)} = \frac{-1000\pm\sqrt{-7\times10^6}}{2} = -500 \pm 500\sqrt{-7}$$

$$= -500 \pm i1323$$

Roots are complex and conjugate

$$C.F(q_c) = e^{-500t}(c_1\cos1323t + c_2\sin1323t)$$

$$P.I(q_p) = \frac{1}{f(D)}Q(t) = \frac{1}{D^2+1000D+2\times10^6}\cdot 0 = 0$$

The G.S of eq^n (1) is $q = q_c + q_p$

$$q = (c_1\cos1323t + c_2\sin1323t)\cdot e^{-500t}$$

$$q = e^{-500t}(c_1\cos1323t + c_2\sin1323t) \rightarrow (2)$$

$$\frac{dq}{dt} = i = e^{-500t}\left(c_1(-1323\sin1323t) + c_2(1323\cos1323)\right) + (c_1\cos1323t +$$

$$c_2\sin1323t)(-500e^{-500t}) \rightarrow (3)$$

G.T when $t=0$, Change $q=0.002C$ & current $i=\frac{dq}{dt}=0$

when $t=0$, $q=0.002$ in (2) we get

$$0.002 = 1\left(c_1(1)+c_2(0)\right)$$

$$c_1 = 0.002$$

when $t=0$, $i=\frac{dq}{dt}=0$ in (3)

$$0 = 1\left(-1323\,c_1(0)+c_2(1323)(1)\right) - 500(1)\left(c_1(1)+c_2(0)\right)$$

$$1323\,c_2 - 500c_1 = 0$$

$$1323\,c_2 = 500c_1$$

$$c_2 = \frac{500c_1}{1323} = \frac{500\times 0.002}{1323} = 7.5585\times 10^{-4}$$

$$= 0.00075585$$

$$c_2 = 0.0008$$

Sub c_1, c_2 in eqn (2)

$$q = e^{-500t}\left(0.002\cos 1323t + 0.0008\sin 1323t\right)$$

B] Solve $(D+6)y - Dx = 0$; $(3-D)x - 2Dy = 0$

Given $(D+6)y - Dx = 0 \rightarrow (1)$

$(3-D)x - 2Dy = 0 \rightarrow (2)$

multiply (1) with $2D \rightarrow -2D^2x + (D+6)2Dy = 0 \rightarrow (3)$

multiply (2) with $D+6 \rightarrow (3-D)(D+6)x - 2D(D+6)y = 0 \rightarrow (4)$

from (3) & (4)
$$2D(D+6)y - 2D^2x = 0$$
$$-2D(D+6)y + (3-D)(D+6)x = 0$$

$$-2D^2x + (3D+18-D^2-6D)x = 0$$

$$-2D^2x + 3Dx + 18x - D^2x - 6Dx = 0$$

$$-3D^2x - 3Dx + 18x = 0$$

$$3D^2x + 3Dx - 18x = 0$$

$D^2x + Dx - 6x = 0$

$(D^2 + D - 6)x = 0$

A.E is $f(m) = 0$

$m^2 + m - 6 = 0$ $\quad -6 \to +3, +2$

$m^2 + 3m - 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$(m+3)(m-2) = 0$

$m = -3, +2$ Roots are real & distinct

$C.F(x) = c_1 e^{2t} + c_2 e^{-3t}$

Sub CF in ② $\quad (3-D)(c_1 e^{2t} + c_2 e^{-3t}) - 2Dy = 0$

$3c_1 e^{2t} + 3c_2 e^{-3t} - D(c_1 e^{2t} + c_2 e^{-3t}) = 2Dy$

$3c_1 e^{2t} + 3c_2 e^{-3t} - c_1 2e^{2t} + 3c_2 e^{-3t} = 2Dy$

$y = \frac{1}{2D}\left[c_1 e^{2t} - 6c_2 e^{-3t}\right]$

$= \frac{1}{2}\left(c_1 \frac{e^{2t}}{2} - \cancel{6}^{2} c_2 \frac{e^{-3t}}{\cancel{3}}\right)$

$y = \frac{c_1}{4} e^{2t} - c_2 e^{-3t}$