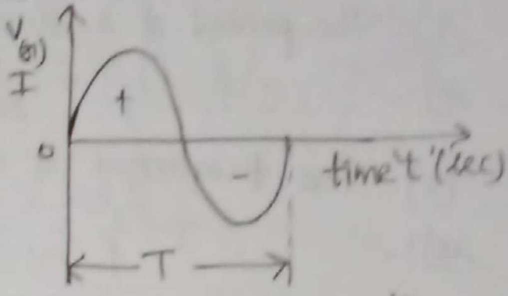
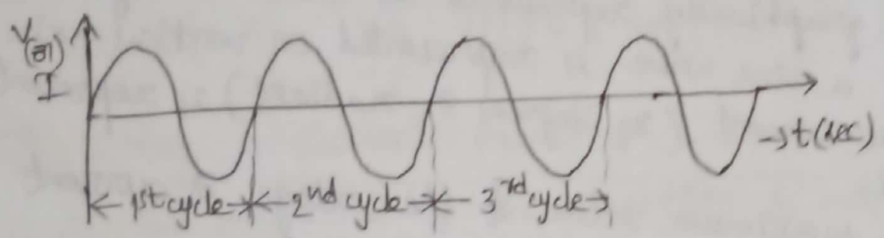
Principle of AC Voltages:

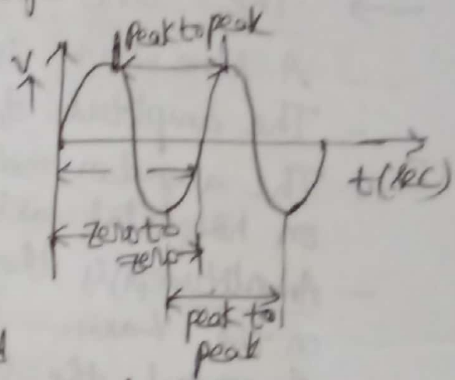
- Mostly alternating voltages and currents are represented by a sinusoidal wave or a sinusoid.
- Basically an alternating voltage (current) waveform is defined as the voltage (current) that varies (fluctuates) with time periodically, with change in polarity and direction.
- A sinusoidal function is easy to analyse, to generate & it is more useful in the power industry.
- As seen from the fig①, the wave changes its magnitude & direction with time.



fig① sinusoidal waveform



fig②



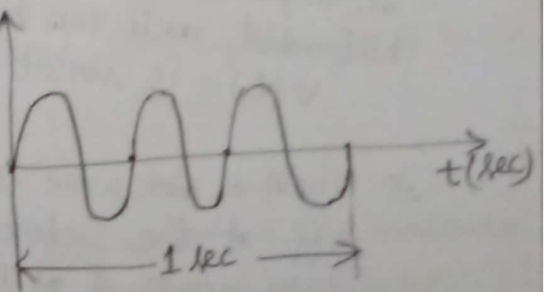
fig③

- If we start at time $t=0$, the wave goes to a maximum value and returns to zero, and then decreases to a negative maximum value before returning to zero. The sine wave changes with time in an orderly manner.
- During the positive portion of voltage, the current flows in one direction, & during the negative portion of voltage, the current flows in opposite direction. The complete +ve and -ve portion of the wave is one cycle of the sine wave. Time is designated by 't'.

* The time taken for any wave to complete one full cycle is called the period (T). In general, any periodic wave constitutes a no. of such cycles. Ex:- One cycle of a sine wave repeats a no. of times as shown in fig②.

- In fig④, the sine wave completes three cycles in one sec.

* Frequency is measured in hertz. 1 hertz is equivalent to one cycle per second, 60 hertz is 60 cycles per second & so on. In fig④, frequency denoted by 'f' is 3Hz, ie., three cycles per second. The relation between time period & frequency is given by $\boxed{f = \frac{1}{T}}$. A sine wave with a longer period consists of fewer cycles than one with a shorter period.

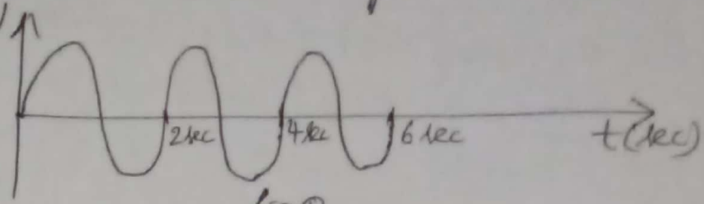


fig④

(1) What is the period of sine wave shown in fig①.

sol:- It can be seen that the sine wave takes two seconds to complete one period in each cycle.

$\therefore T = 2\,sec$



fig①

(2) The period of a sine wave is 20ms. What is the frequency?

sol:- $\therefore f = \frac{1}{T} = \frac{1}{20ms} = 50\,Hz$.

(3) The frequency of a sine wave is 30 Hz. What is its period?

sol:- $\therefore T = \frac{1}{f} = \frac{1}{30} = 0.03333\,sec$

$= 33.33\,msec$

→ The Sine wave equation :-

- A sine wave is graphically represented as shown in fig 1(a) below.
- The amplitude of a sine wave is represented on vertical axis. The angular measurement (in degrees or radians) is represented on horizontal axis.
- Amplitude(A) is the maximum value of the voltage or current on the Y-axis.
- In general, the sine wave is represented by the equation, $\boxed{v(t) = V_m \sin\omega t}$ —①

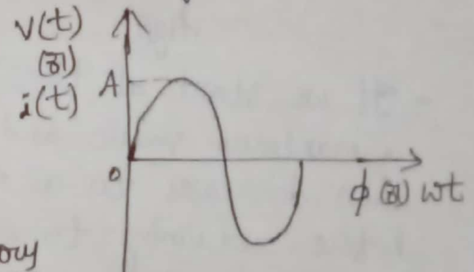


fig 1(a)

- The eq① states that any point on the sine wave represented by an instantaneous value v(t) is equal to the maximum value times the sine of the angular frequency at that point.

Ex: If A certain sine wave vtg has peak value of 20V, the instantaneous voltage at a point π/4 radians along the horizontal axis can be calculated as,

$v(t) = V_m \sin\omega t = 20 \sin(\frac{\pi}{4})$

$= 20 \times 0.707 = 14.14V$

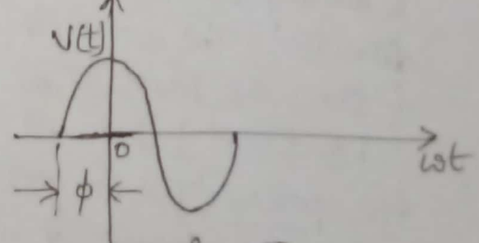


fig 1(b)

* When a sine wave is shifted to the left of the reference wave by a certain angle φ as shown in fig 1(b), the general expression can be written as,

$\boxed{\therefore v(t) = V_m \sin(\omega t + \phi)}$ —②

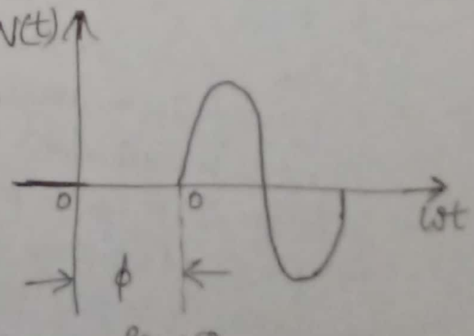
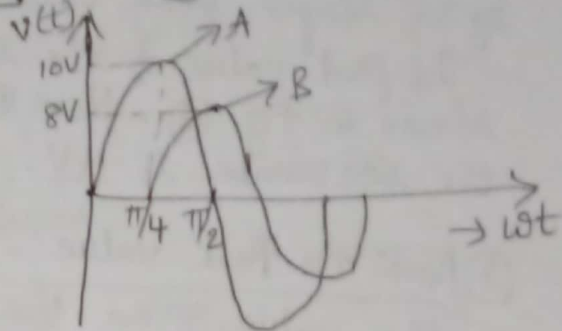


fig 1(c)

When a sine wave is shifted to the right of the reference wave by a certain angle ϕ, as shown in fig 1©, the general expression is $\boxed{\therefore V(t) = V_m \sin(\omega t - \phi)}$ —— ③



Ex: ① Determine the instantaneous value at the 90° point on the x-axis for each sine wave shown in the fig.

Sol:- From the fig., the equation for the sine wave 'A' is $v(t) = 10 \sin \omega t$

The value at $\pi/2$ in this wave is $V(t) = 10 \sin \frac{\pi}{2}$
$= 10V$

The equation for sine wave 'B' is
$v(t) = 8 \sin(\omega t - \frac{\pi}{4})$

At $\omega t = \pi/2$ → $v(t) = 8 \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 8 \sin 45°$
$= 8(0.707) = 5.66V$

→ Voltage & Current values of a Sine wave :-

As the magnitude of the waveform is not constant, the waveform can be measured in different ways. These are :

① Instantaneous value
② Peak value
③ Peak to peak value
④ Root Mean Square (RMS) value
⑤ Average value

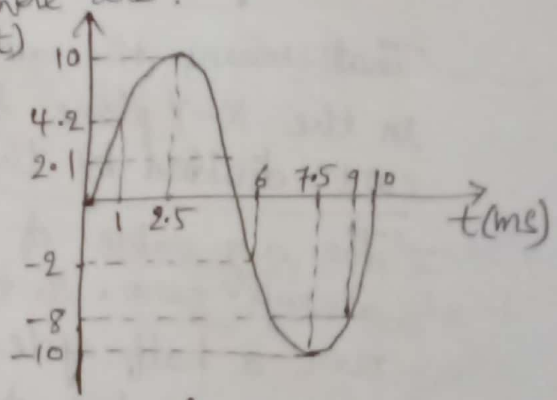


fig 1ⓐ

① Instantaneous value

Consider the sine wave shown in fig 1ⓐ.

At any given time, it has some instantaneous value. This value is different at different points along the waveform.

In fig. 1ⓐ, during the positive cycle, the instantaneous values are positive & during the negative cycle, the instantaneous values are negative. It is shown at time 1ms → the value is 4.2V

At 2.5 ms, the value is 10V
At 6 ms → " " is ~~—~~ −2V
At 7.5ms → " " " −10V

② Peak value :- The peak value of the sine wave is the maximum value of the wave during positive half cycle, or maximum value of the wave during negative half cycle.

Since the value of these two are equal in magnitude, a sine wave is characterised by a single peak value.

- The peak value of the sine wave is shown in fig. 1(b). Here the peak value of the sine wave is '4V'.

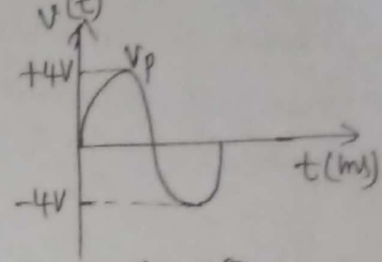


fig. 1(b)

③ Peak to Peak value :- The peak to peak value of a sine wave is the value from the positive peak to the negative peak as shown in fig 1(b). Here the peak to peak value is '8V'.

④ Average value :- (Avg)

The average value of any function $v(t)$, with period 'T' is given by,

$$\therefore V_{(avg)} = \frac{1}{T}\int_0^T v(t)\, dt$$

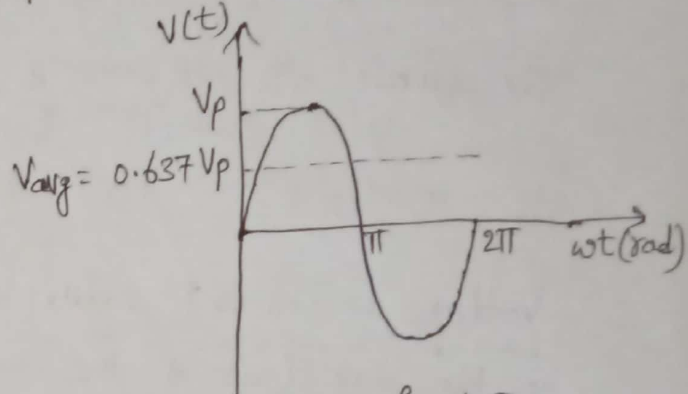


fig 1(c)

That means the average value of a curve in the X-Y plane is the total area under the complete curve divided by the distance of the curve.

- The avg value of a sine wave over one complete cycle is always zero. So the avg value of a sine wave is defined over a half cycle and not a full cycle period.

* The avg value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

$$\boxed{\therefore \text{Avg value of sine wave} = \frac{\text{Total area under half-cycle curve}}{\text{Distance of the curve}}}$$

The avg value of the sine wave $v(t) = V_p \sin \omega t$ is given by,

$$V_{(avg)} = \frac{1}{\pi}\int_0^{\pi} V_p \sin \omega t \, d(\omega t)$$

$\cos \pi = -1$

$$= \frac{V_p}{\pi}\left[-\cos \omega t\right]_0^{\pi} = \frac{V_p}{\pi}\left[-(-1-1)\right]$$

$$= \frac{2V_p}{\pi} = 0.637\, V_p$$

$$\boxed{\therefore V_{(avg)} = 0.637\, V_p}$$

⑤ Root Mean Square value (or) Effective value

The root mean square (rms) value of a sine wave is a measure of the heating effect of the wave.

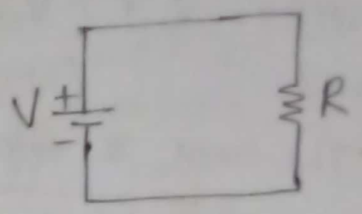


fig 1ⓐ

- When a resistor is connected across a dc vtg source as shown in fig. 1ⓐ, a certain amount of heat is produced in the resistor in a given time.

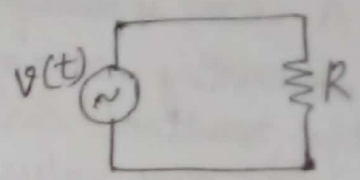


fig 1ⓑ

- A similar resistor is connected across an ac vtg source for the same time as shown in fig. 1ⓑ. The value of the ac vtg is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. This value is called the rms value
- That means the rms value of a sine wave is equal to the dc vtg that produces the same heating effect.
- In general, the rms value of any function with period 'T' has an effective value given by, $V_{rms} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2\, dt}$

Consider a function $v(t) = V_p \sin\omega t$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_p \sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \sqrt{\frac{V_p^2}{2\pi}\cdot\frac{1}{2}\left[\int_0^{2\pi} 1.\, d\omega t - \int_0^{2\pi}\cos 2\omega t .\, d\omega t\right]}$$

$$= \sqrt{\frac{V_p^2}{4\pi}\left\{[\omega t]_0^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{2\pi}\right\}}$$

$$= \sqrt{\frac{V_p^2}{4\pi}\left\{[2\pi - 0] - \frac{1}{2}\underbrace{[\sin 2(2\pi) - \sin 2(0)]}_{0}\right\}}$$

$$= \sqrt{\frac{V_p^2}{4\pi_2}.(2\pi)} = \frac{V_p}{\sqrt{2}} = 0.707\, V_p$$

$$\boxed{\therefore V_{rms} = 0.707\, V_p}$$

$$\left[\because \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x\right]$$

$$\sin^2\omega t = \frac{1-\cos 2\omega t}{2}$$

$$\cos 2x = 1 - 2\sin^2 x$$
$$2\sin^2 x = 1 - \cos 2x$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\left[\because \int \cos 2x = \frac{\sin 2x}{2}\right]$$

If the function consists of a no. of sinusoidal terms, that is
$v(t) = V_0 + (V_{c1}\cos\omega t + V_{c2}\cos 2\omega t + \ldots) + (V_{s1}\sin\omega t + V_{s2}\sin 2\omega t + \ldots)$

The rms, ~~a~~ effective value is given by,

$$\therefore V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_{c1}^2 + V_{c2}^2 + \ldots) + \frac{1}{2}(V_{s1}^2 + V_{s2}^2 + \ldots)}$$

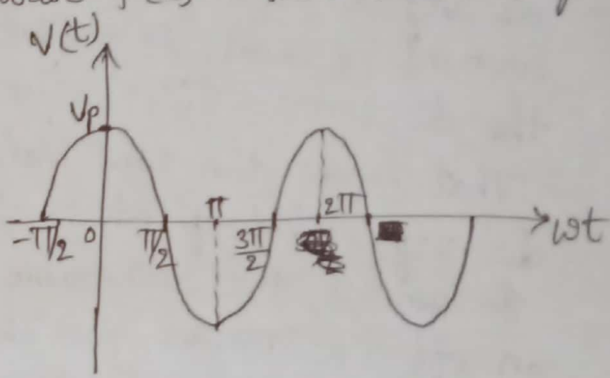
Ex: ① A wire is carrying a direct current of 20A and a sinusoidal alternating current of peak value 20A. Find the rms value of the resultant current in the wire.

Sol:- The rms value of the combined wave,

$$= \sqrt{20^2 + \frac{20^2}{2}} = \sqrt{400 + 200} = \sqrt{600} = 24.5A$$

② Find the average value of a cosine wave $f(t) = \cos\omega t$ shown in fig.

Sol:- Average value of a cosine wave is $v(t) = V_p \cos\omega t$



$$V_{avg} = \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} V_p \cos\omega t \, d(\omega t)$$

$$= \frac{1}{\pi} \cdot V_p \left[-\sin\omega t\right]_{\pi/2}^{3\pi/2}$$

$$= \frac{-V_p}{\pi}[-1-1] = \frac{2V_p}{\pi} = 0.637\, V_p$$

→ **Peak Factor**:- The peak factor of any waveform is defined as the ratio of the peak value of the wave to the rms value of the wave.

$$\boxed{\therefore \text{Peak Factor} = \frac{V_p}{V_{rms}}}$$

$\therefore$ Peak factor of the sinusoidal waveform $= \dfrac{V_p}{V_p/\sqrt{2}} = \sqrt{2} = 1.414$

→ **Form Factor**:- Form factor of a waveform is defined as the ratio of rms value to the avg value of the wave.

$$\boxed{\therefore \text{Form factor} = \frac{V_{rms}}{V_{avg}}}$$

Form factor of a sinusoidal waveform $= \dfrac{V_p/\sqrt{2}}{0.637 V_p}$

$= 1.11$

$\frac{V_p}{\sqrt{2}} \times \frac{1}{(0.637 V_p)}$

$= \frac{1}{\sqrt{2} \times 0.637}$

$= 1.11$

(1) A sine wave has a peak value of 12V. Determine the following values: ⓐ rms ⓑ avg ⓒ crest factor ⓓ F.F ⓔ Peak to peak

Sol :- ⓐ rms value of the given sine wave, $V_{rms} = 0.707\,V_p$

$$= 0.707 \times 12 = 8.48\text{V}$$

ⓑ Avg value of sine wave, $V_{avg} = 0.637\,V_p$

$$= 0.637 \times 12 = 7.64\text{ V}$$

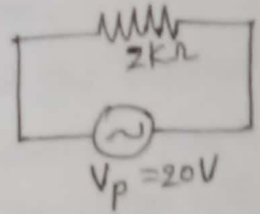
ⓒ Crest (or) peak factor of sine wave $= \dfrac{\text{Peak value}}{\text{rms value}} = \dfrac{12}{8.48} = 1.415$

ⓓ Form factor (F.F) $= \dfrac{\text{rms value}}{\text{avg value}} = \dfrac{8.48}{7.64} = 1.11$

ⓔ peak to peak value of sine wave, $V_{pp} = 2V_p = 2 \times 12 = 24\text{ V}$

(2) A sinusoidal vtg is applied to the resistive ckt shown in fig. Determine the following values:

ⓐ I_{rms} ⓑ I_{avg} ⓒ I_p ⓓ I_{pp}



Sol :- The function given to the ckt shown is,

$$V(t) = V_p \sin\omega t = 20 \sin\omega t$$

The current passing through resistor is $i(t) = \dfrac{V(t)}{R}$

$$i(t) = \frac{20}{2 \times 10^3} \sin\omega t = 10 \times 10^{-3} \sin\omega t$$

$$\therefore I_p = 10 \times 10^{-3} \text{ Amps}$$

The peak value $I_p = 10\text{mA}$

Peak to peak value $I_{pp} = 2I_p = 2 \times 10\text{mA} = 20\text{ mA}$

rms value $I_{rms} = 0.707 I_p = 0.707 \times 10\text{mA} = 7.07\text{ mA}$

average value $I_{avg} = 0.637\,I_p = 0.637 \times 10\text{mA} = 6.37\text{ mA}$

Network Theorems: (applied to analyze electrical & electronic ckts)

Thevenin's Theorem

- It is applicable where it is desired to determine the current through (or) voltage across any element in a network without going through the rigorous method of solving a set of n/w eqs.

Statement:-
"Any two terminal bilateral linear d.c ckts can be replaced by an equivalent ckt consisting of a voltage source and a series resistor."

Explanation:- ① Let us consider a simple d.c ckt as shown in fig. 1(a). Now find 'I_L' by Thevenin's theorem.

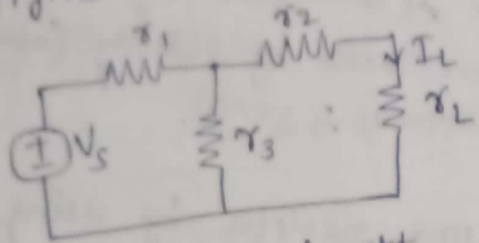


fig. 1(a) A simple d.c ckt

② In order to find the eqt vtg source, r_L is removed and $V_{o.c}$ is shown in fig. 1(b) and $V_{o.c}$ is calculated.

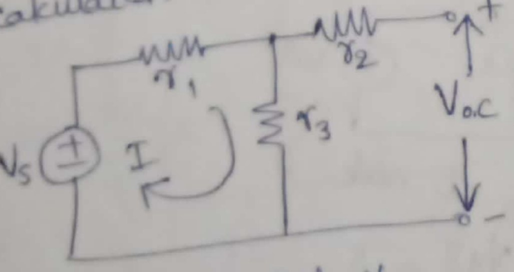


fig. 1(b) Finding of $V_{o.c}$

$$\therefore V_{o.c} = I r_3 \quad (\because r_2 = 0)$$

$$I = \frac{V_s}{r_1 + r_3}$$

$$\boxed{\therefore V_{oc} = \frac{V_s}{r_1 + r_3} \cdot r_3}$$

③ Next to find the internal resistance of the n/w (Thevenin's resistance or eqt. resistance) in series with $V_{o.c}$, the vtg source is removed (deactivated) by a short ckt (as the source does not have any internal resistance) as shown in fig. 1(c).

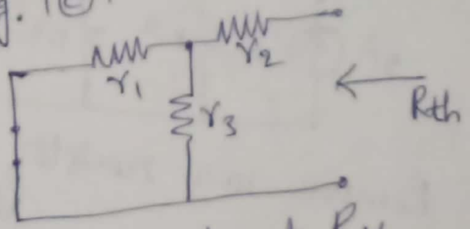


fig. 1(c) Finding of R_{th}

$$\boxed{\therefore R_{th} = r_2 + \frac{r_1 r_3}{r_1 + r_3}}$$

④ Finding of 'I_L' from Thevenin's eqt ckt, shown in fig. 1(d)

$$\boxed{\therefore I_L = \frac{V_{o.c}}{R_{th} + R_L}}$$

Equivalent ckt

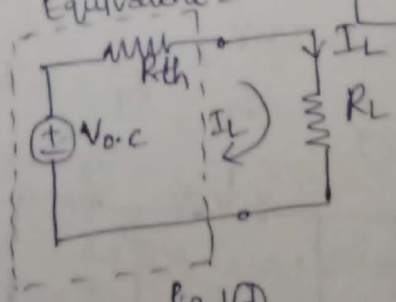


fig. 1(d)

Steps for solving a n/w using Thevenin's theorem:-

Step ①: Remove the load resistance (R_L) and find the open ckt voltage ($V_{o.c}$) across the open ckted load terminals.

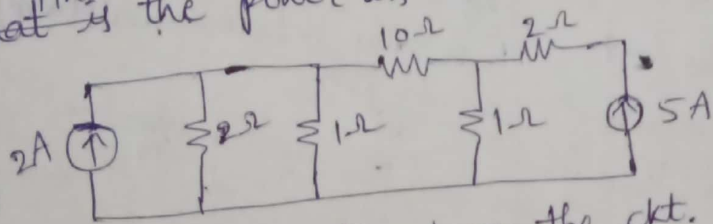
Step ②: Deactivate the constant sources (for vtg source, remove it by internal resistance and for current source delete the source by open ckt) and find the internal resistance (Thevenin's resistance) of the source side looking through the open ckted load terminals. Let this resistance be R_{th}.

Step ③: Obtain Thevenin's eqt ckt by placing R_{th} in series with V_{oc}

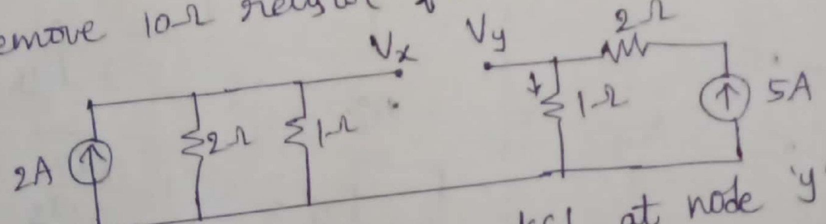
Step ④: Reconnect 'R_L' across the load terminals as shown in fig. 1(d)

To Find R_{th}: (a) Independent current source is deactivated by removing source.
" vtg source is deactivated by shorting (assuming its internal impedance (resistance) is zero).

(1) Find the power loss in the 10Ω resistor by using Thevenin's theorem.



Sol):- Remove 10Ω resistor from the ckt.



KCL at node 'x':

$$\frac{V_x}{1} + \frac{V_x}{2} = 2$$

$V_x = 1.33\text{ V}$

KCL at node 'y'

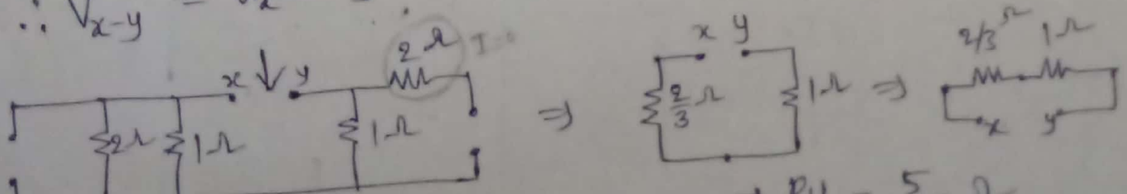
$$\frac{V_y}{1} = 5$$

$V_y = 5\text{ V}$

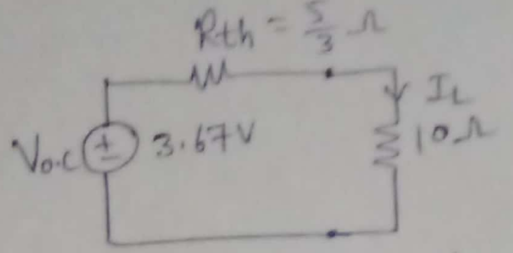
$\therefore V_{x-y} = V_x - V_y = 1.33 - 5 = -3.67\text{ V} = V_{o.c}$

$\frac{2}{3}+1 = \frac{2+3}{3} = 5/3$

$\frac{2\times1}{2+1} = \frac{2}{3}$



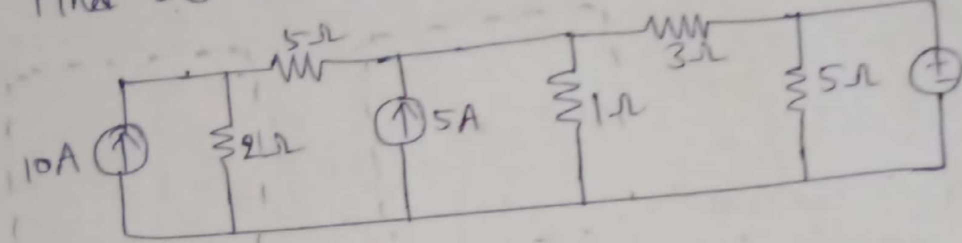
$\Rightarrow R_{th} = \frac{5}{3}\,\Omega$



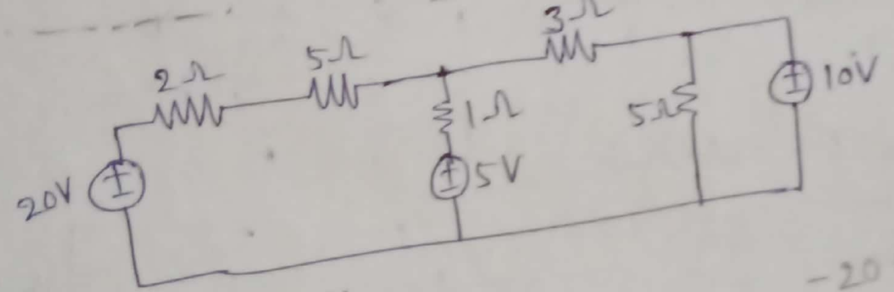
$$I_L = I_{10\Omega} = \frac{V_{oc}}{R_{th}+R_L} = \frac{3.67}{\frac{5}{3}+10} = 0.315\ A$$

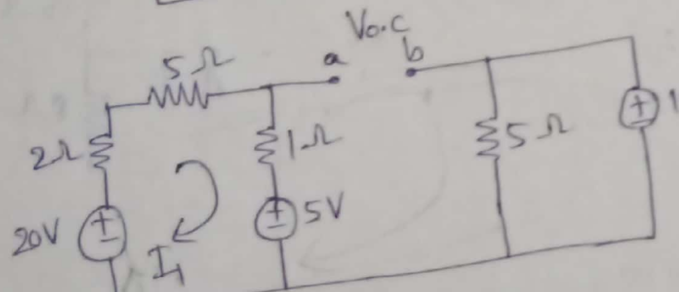
$$\therefore P = I^2_{10\Omega} R = (0.315)^2 (10)$$
$$= 0.99225\ W$$

(2) Find the current in 3Ω resistor for the ckt shown.



sol:-



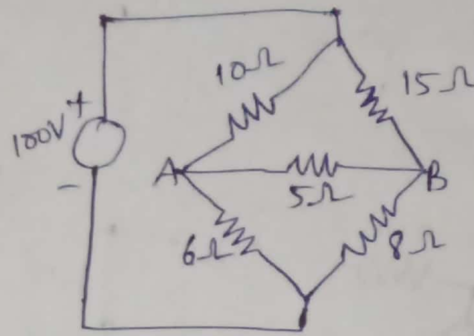


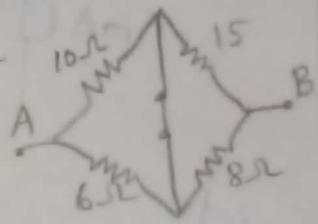
$$-20 + 2I_1 + 5I_1 + I_1 + 5 = 0$$
$$8I_1 = 15$$
$$I_1 = \frac{15}{8}$$

$$I_1 = \frac{20-5}{2+5+1} = \frac{15}{8}\ A$$

$$V_{oc} = 20 - \frac{15}{8}(2+5) - 10$$
$$= -3.125\ V$$

→ Ex: Use Thevenin's theorem to find the current through 5Ω resistor.

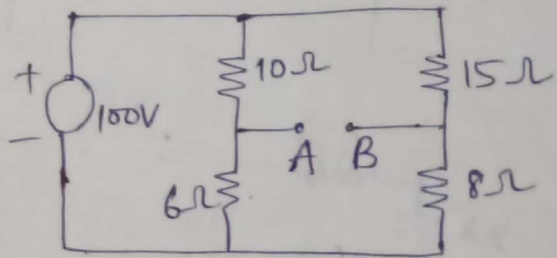




sol:-

Current in 6Ω resistor is

$$I_{6\Omega} = \frac{100}{10+6} = \frac{100}{16} = 6.25A$$



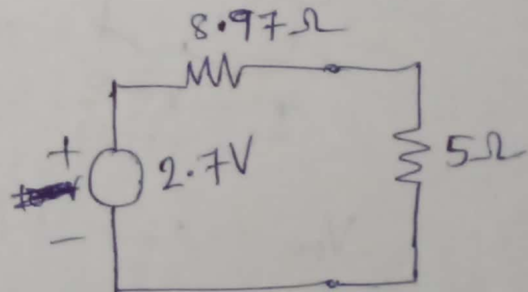
Vtg across 6Ω resistor is

$$V_{6\Omega} = 6.25 \times 6 = 37.5V$$

$$I_{8\Omega} = \frac{100}{8+15} = 4.35A$$

$$V_{8\Omega} = 4.35 \times 8 = 34.8V$$

$$\therefore V_{AB} = 37.5 - 34.8 = 2.7V$$



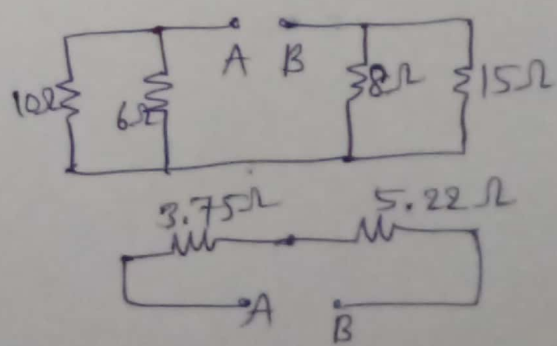
$$R_{AB} = R_{th} = (6 \| 10) + (8 \| 15)$$

$$= \frac{6 \times 10}{6+10} + \frac{8 \times 15}{8+15}$$

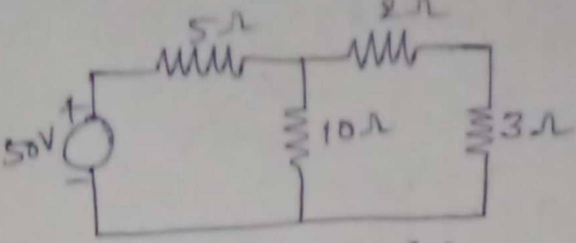
$$= 3.75 + 5.22 = 8.97\Omega$$

∴ Current in 5Ω resistor is

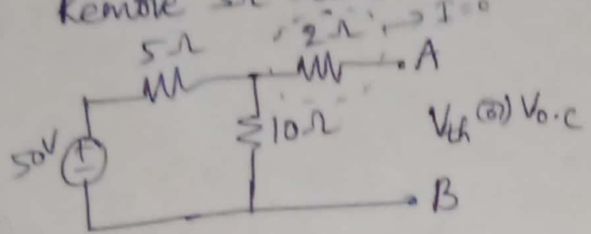
$$\therefore I_{5\Omega} = \frac{2.7}{8.97+5} = 0.193A$$

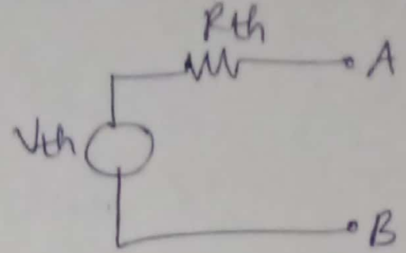


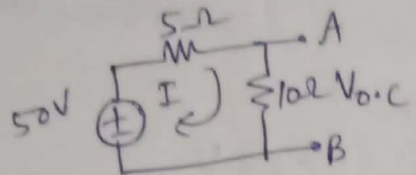
1. Use Thevenin's theorem to find the current in 3Ω resistor shown in fig.

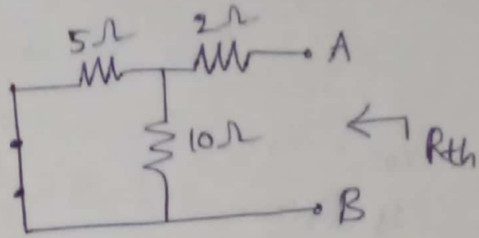


Sol:- Remove 3Ω resistor









$V_{th} = V_{o.c} = V_{AB} = IR = I(10)$

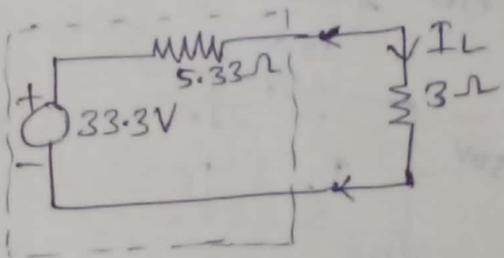
where $I = \frac{V}{R_{eqt}} = \frac{50}{5+10} = \frac{50}{15}$

$\therefore V_{th} = \frac{50}{15} \times 10 = 33.3\ V$

$R_{th} = (5 \| 10) + 2$

or $R_{AB} = \frac{5 \times 10}{5+10} + 2$

$= 5.33\ \Omega$

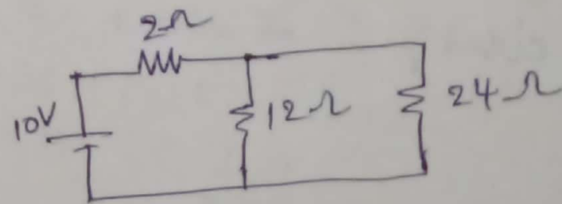


$\therefore I_L = \frac{V_{th}}{R_{th} + R_L}$

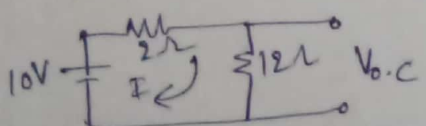
$= \frac{33.3}{5.33+3}$

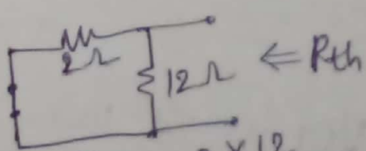
$= 4.00\ A$

(2) Find the current through 24Ω resistor using Thevenin's theorem for the ckt shown.



Sol:- Remove 24Ω resistor



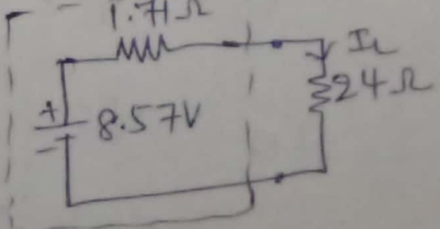


$R_{th} = \frac{2 \times 12}{2+12} = \frac{24}{14}$

$= 1.71\ \Omega$

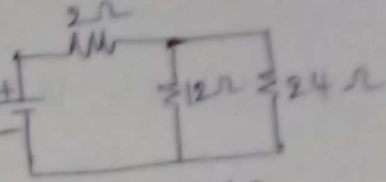
$I = \frac{V}{R_{eqt}} = \frac{10}{2+12} = \frac{10}{14}$

$\therefore V_{o.c} = I.R = I(12) = \frac{10}{14} \times 12 = 8.57V$



$\therefore I_L = \frac{8.57}{1.71+24} = 0.33A$

Common method :-



$$I_{24\Omega} = I_T \times \frac{12}{12+24}$$

where $I_T = \frac{V}{R_{eqt}}$

$$= \frac{10}{10} = 1\,A$$

$$R_{eqt} = (12 \| 24) + 2$$
$$= \frac{\cancel{12} \times \cancel{24}^{8}}{\cancel{36}_{1}} + 2$$
$$= 8 + 2 = 10\,\Omega$$

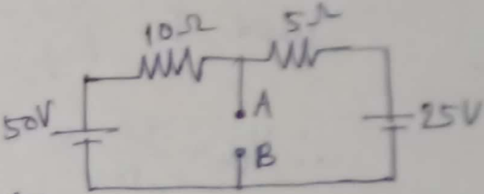
$$\therefore I_{24\Omega} = 1 \times \frac{12}{12+24} = \frac{12}{36} = \frac{1}{3} = \underline{\underline{0.33A}}$$

$$\therefore V_{24\Omega} = I_{24\Omega} \times 24 = 0.33 \times 24 = 7.92\,V$$

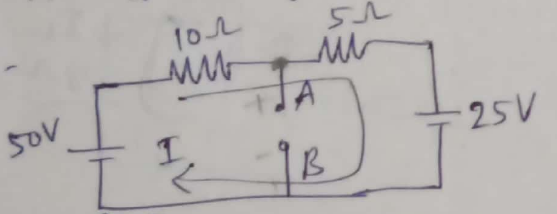
$V_{oc} = V_{AB}$ = Vtg across 12 Ω resistor when load resistance (24 Ω) is disconnected from the ckt then V_{th} is

(Vtg division rule) $V_{th} = 10 \times \frac{12}{2+12} = 10 \times \frac{12}{14} = \underline{\underline{8.57V}}$

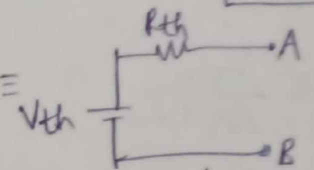
(3) Determine the Thevenin's eqt ckt across 'AB' for the given ckt shown in fig.



Sol :-



fig①



To solve for V_{th}, we have to find the vtg drops around the closed path as shown in above fig①.

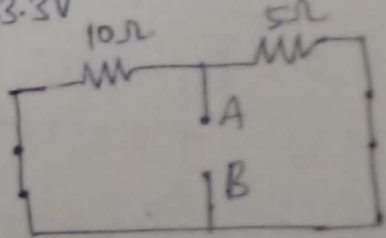
$$-50 + 10I + 5I + 25 = 0$$
$$15I - 25 = 0$$
$$15I = 25$$
$$I = \frac{25}{15} = 1.67A$$

$$50 - 25 = 10I + 5I$$
$$15I = 25$$
$$I = 1.67A$$

Vtg across 10Ω is $V_{10\Omega} = 1.67 \times 10 = 16.7V$

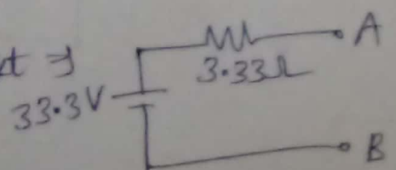
vtg drop across 5Ω is $V_{5\Omega} = 1.67 \times 5 = 8.35V$

$$V_{th} = V_{AB} = 50 - V_{10} = 50 - 16.7 = 33.3V$$



$$R_{th} = 10 \| 5 = \frac{10 \times 5}{10+5} = 3.33\,\Omega$$

thevenin's eqt ckt is



Norton's Theorem:- It is converse of Thevenin's theorem.

Statement:- "A linear active network consisting of independent and a dependent voltage & current sources and linear bilateral network elements can be replaced by an eqt ckt consisting of a current source in parallel with a resistance, the current source being the short ckted current across the load terminal and the resistance being the internal resistance of the source n/w looking through the open ckted load terminals."

Explanation: ① In order to find the current through r_L, the load resistance (fig. 1(a)), by Norton's theorem, let us replace 'r_L' by short ckt (fig. 1(b)).

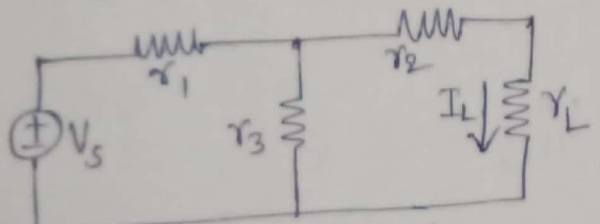


fig. 1(a). A simple dc n/w

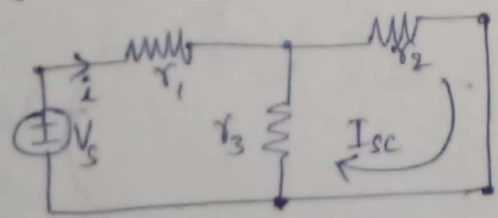


fig. 1(b) Finding of i_{sc}

obviously, $i = \dfrac{V_s}{r_1 + \dfrac{r_2 r_3}{r_2 + r_3}}$

$$\boxed{\therefore\ i_{sc} = i \times \frac{r_3}{r_3 + r_2}}$$

② Next, the short ckt is removed and the independent source is deactivated as done in Thevenin's theorem. (fig. 1(c)).

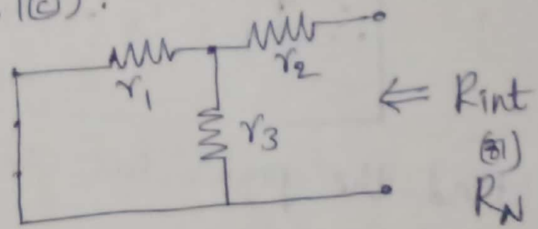


fig. 1(c). Finding of R_{int}

③ As per Norton's theorem, the eqt source ckt would contain a current source in parallel to the internal resistance, the current source being the short ckted current across the shorted terminals of the load resistor (fig. 1(d)).

Eqt source n/w

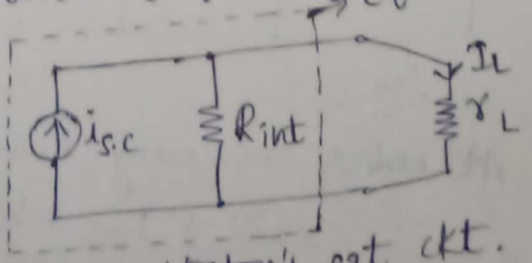


fig. 1(d). Norton's eqt ckt.

Note: Finding R_{int} for source system is same in both Norton's & Thevenin's theorems.

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Steps for solving a network utilizing Norton's theorem:

Step ①: Remove the load resistor and find the internal resistance of the source n/w by deactivating the constant sources. This procedure is same as described for Thevenin's theorem. Let this resistance be 'R_{int}'.

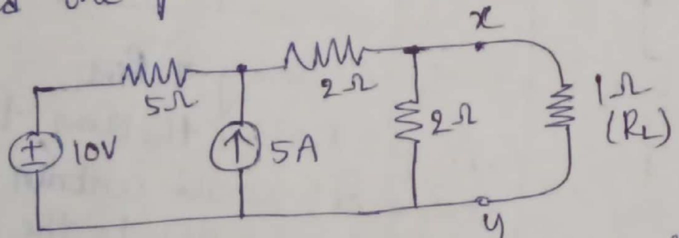
Step ②:- Next, short the load terminals and find the short ckt current flowing through the shorted load terminals using conventional n/w analysis. Let this current be 'i_{sc}'.

Step ③:- Norton's eqt ckt is drawn by keeping R_{int} in parallel to i_{sc} as shown in fig. 1ⓐ

Step ④:- Reconnect the load resistor (R_L) across the load terminals and the current through it (I_L) is then given by,

$$\boxed{\therefore I_L = i_{sc} \cdot \frac{R_{int}}{R_{int} + R_L}}$$

(1) Find the power loss in 1Ω resistor (R_L) using Norton's theorem.



Sol:- Let us remove 1Ω resistor & short x-y shown in fig. 1ⓐ

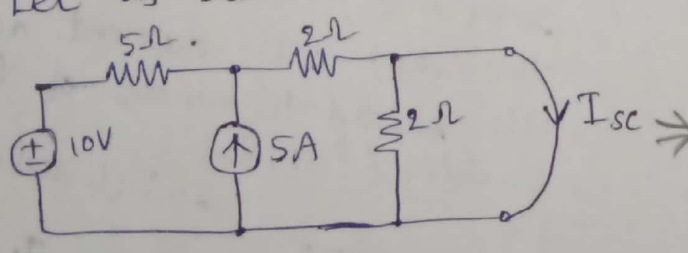


fig. 1ⓐ

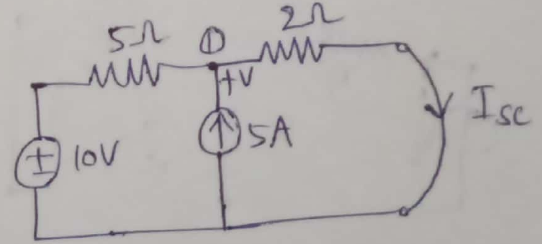


fig. 1ⓑ

At node ①, assuming node potential to be 'V',

$$\frac{V-10}{5} + I_{sc} = 5 \quad — ①$$

But $I_{sc} = \frac{V}{2}$ (fig. 1ⓑ)

$\therefore$ from ①, $\frac{V-10}{5} + \frac{V}{2} = 5 \Rightarrow 0.7V = 7 \Rightarrow V = 10\,V$

ie., $\therefore I_{sc} = \frac{V}{2} = \frac{10}{2} = 5A$

To find R_{int}, all constant sources are deactivated as shown in fig. 1ⓒ where $R_{int} = \frac{(5+2)\times 2}{5+2+2} = 1.56\Omega$

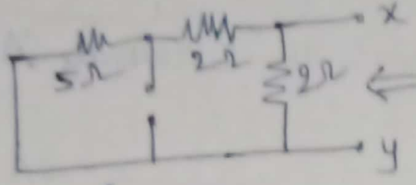


fig. 1ⓒ

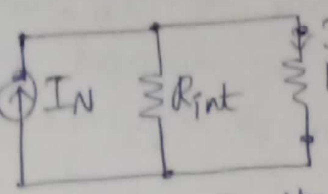


fig. 1ⓓ Norton's eqt ckt

$\therefore R_{int} = 1.56\Omega$

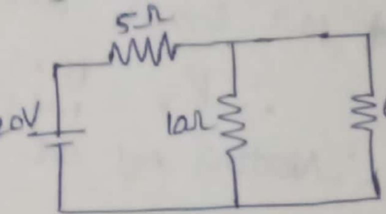
$I_{sc} = I_N = 5A$

Here, $I = I_N \cdot \frac{R_{int}}{R_{int}+1} = 5 \times \frac{1.56}{1.56+1} = 3.04A$.

$\therefore$ Power loss in 1Ω resistor $= I^2R$

$= (3.04)^2(1) = 9.26\ W$.

(2) Determine the current through 6Ω resistor using Norton's theorem.



Sol:- Remove the 6Ω resistor & short AB terminals as shown in fig. 1ⓐ

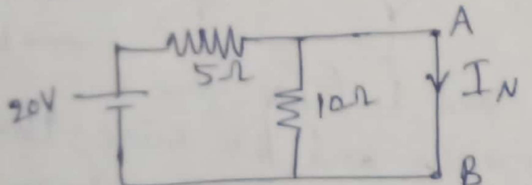


fig. 1ⓐ

Here $I_N = \frac{20}{5} = 4A$

Norton's resistance is equal to the parallel combination of both 5Ω & 10Ω resistors.

$\therefore R_N = \frac{5\times 10}{5+10} = \frac{50}{15} = 3.33\Omega$

$I_{6\Omega} = I_T \times \frac{10}{10+6}$

$I_T = \frac{20}{5+(10\|6)} = 2.285A$

$\therefore I_{6\Omega} = 2.285 \times \frac{10}{16}$

$= 1.43\ A$.

$V_{6\Omega} = I_{6\Omega} \times (6)$

$= 1.43 \times 6$

$= 8.58\ V$

The Norton's eqt source is shown in fig. 1ⓑ

Then current through 6Ω & vtg across it due to Norton's eqt. ckt is,

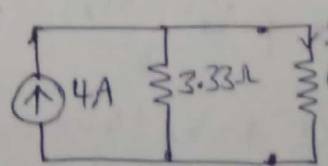


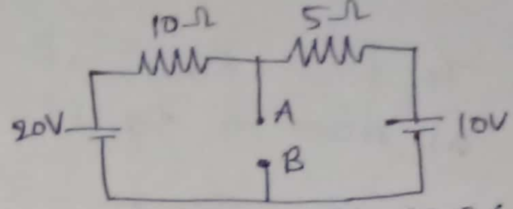
fig. 1ⓑ

$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43A$

$\therefore V_{6\Omega} = 1.43 \times 6 = 8.58V$

thus proved that $R_L (=6\Omega)$ has same values of current & vtg in both the original ckt & Norton's eqt ckt.

(3) Determine Norton's eqt ckt at terminals AB for the ckt shown.



Sol:- To solve for I_N, we have to find the current passing through the terminals 'AB' as shown in fig. 1(a).

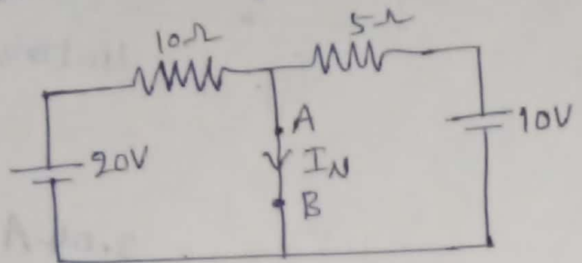


fig. 1(a)

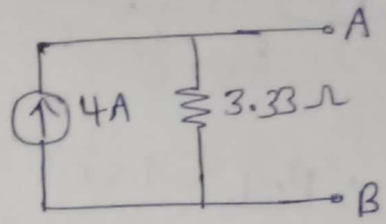


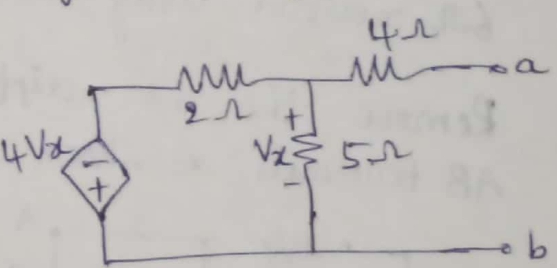
fig. 1(b)

From fig. 1(a), the current passing through the terminals 'AB' is 4A. The resistance at terminals AB is the parallel combination of 10 Ω & 5 Ω resistors.

$$\therefore R_N = \frac{10 \times 5}{10+5} = 3.33\,\Omega$$

∴ Norton's eqt ckt is shown in fig. 1(b).

(4) For the ckt shown in fig, find Norton's eqt ckt.



Sol: In case of ckt having only dependent sources (without independent sources), both V_{oc} and i_{sc} are zero. We apply a 1A source externally and determine the resultant vtg across it, & then find $R_{th} = \frac{V}{1}$ (or) we can also apply the 1V source externally & determine the current through it & then we find $R_{th} = \frac{1}{i}$.

By applying the 1A source externally as shown in fig. 1(a)

Application of KCL, we have

$$\frac{V_x}{5} + \frac{V_x - (-4V_x)}{2} = 1$$

$$\frac{V_x}{5} + \frac{V_x + 4V_x}{2} = 1$$

$$\therefore V_x = 0.37\,V$$

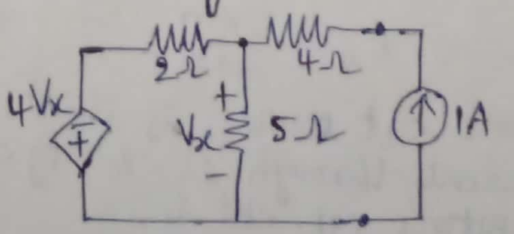


fig. 1(a)

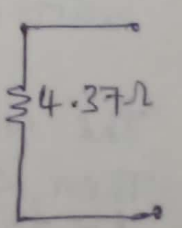


fig. 1(b)

The current in the 4Ω branch is,

$$\frac{V_x - V}{4} = -1$$

Substituting V_x in the above eq. we get,

$$V = 4.37\ V$$

$$\therefore R_{th} = \frac{V}{1} = 4.37\ \Omega$$

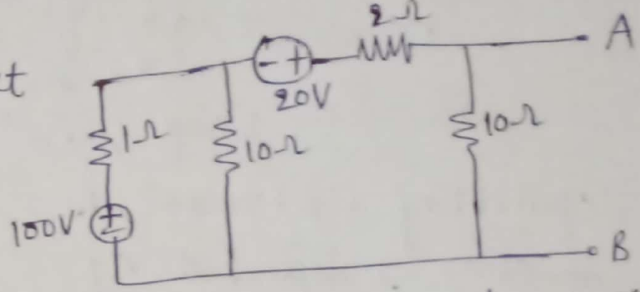
If we short ckt the terminals a and b we have,

$$\frac{V_x - 4V_x}{2} = 0 \Rightarrow V_x = 0$$

$$\therefore I_{sc} = \frac{V_x}{4} = 0$$

$\therefore$ Norton's eqt ckt is as shown in fig 1.(b).

(5) Obtain the Norton's eqt ckt at the terminals AB for the fig. shown.



Sol:-

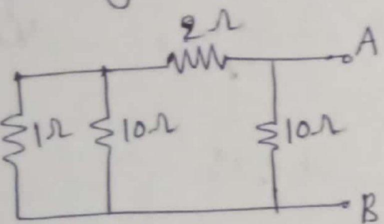


fig. 1(a)

$\Leftarrow R_{eqt}$

For finding the Norton's resistance, replace the vtg sources by the short ckt.

$$\therefore R_{eqt} = \{[(1 \| 10) + 2] \| 10\}$$
$$= 2.253\ \Omega$$

For finding the I_N, short the terminals A and B, find current I_N.

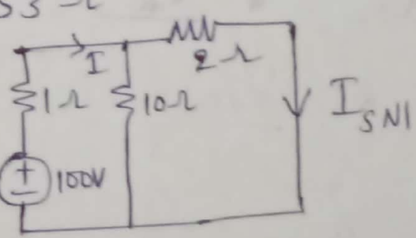


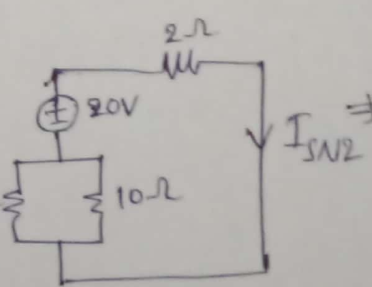
fig. 1(b)

Apply Superposition,

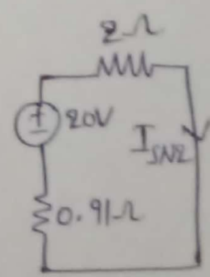
(i) With 100V source,

$$Z = [(2 \| 10) + 1] = 2.67\ \Omega$$

$$I_{SN1} = \frac{100}{Z} = \frac{100}{2.67} = 37.45A$$



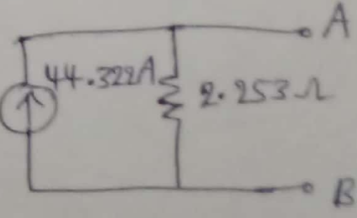
$\Rightarrow$



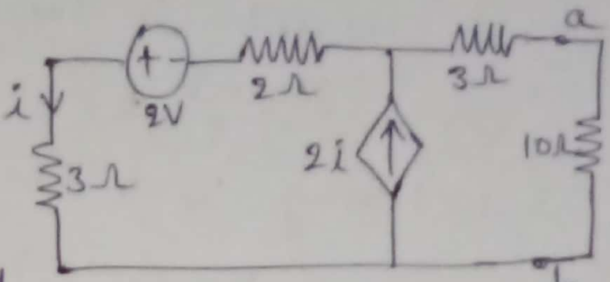
(ii) With 20V source

$$I_{SN2} = \frac{20}{2.91} = 6.872\ A$$

$$\therefore I_{SN} = I_{SN1} + I_{SN2}$$
$$= 37.45 + 6.872 = 44.322A$$



(6) Find the Norton's eqt across the terminals 'ab' as shown in fig. Hence find current through 10Ω.



Sol:- Short circuiting a-b terminals,

$2i = i + i_{sc}$

$\Rightarrow i_{sc} = i$

$i = \frac{2+V}{5} = \frac{V}{3}$

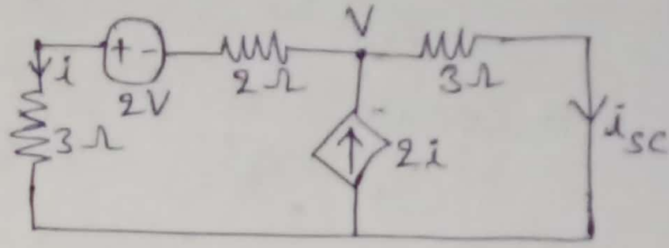


fig. 1(a)

(a) $6+3V = 5V$

$6 = 5V-3V \Rightarrow 6 = 2V$

$\therefore V = \frac{6}{2} = 3V$

$\therefore i = \frac{2+3}{5} = 1$ Amp , $\therefore i_{sc} = 1$ Amp

Open ckting a-b terminal & deacting independent vtg source.

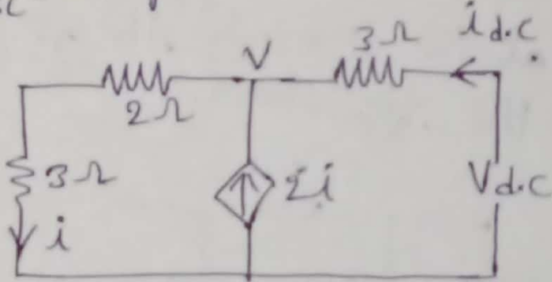


fig. 1(b)

$2i + i_{dc} = i$

$\therefore i_{dc} = -i$

Now, $\frac{V}{5} = i$ (a) $V = 5i$

Now, $\frac{V_{dc} - V}{3} = i_{dc}$

(a) $\frac{V_{dc} - 5i}{3} = -i \Rightarrow V_{dc} - 5i = -3i$

$V_{dc} = 2i$

$\therefore V_{dc} = -2\, i_{dc}$

$V_{dc} = -2 i_{dc}$

$\therefore R_{int} = \frac{V_{dc}}{i_{sc}}$

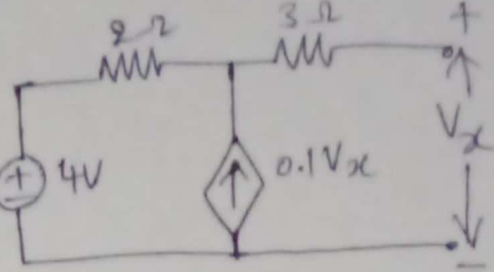
$= \frac{-2 i_{dc}}{1}$

$= \frac{-2(-i)}{1}$

$= \frac{2i}{1} = \frac{2(1)}{(1)} = 2\,\Omega$

$\therefore R_{int} = 2\Omega$

(4) For the ckt shown in fig, obtain thevenin's eqt. ckt.



Sol:- The ckt consists of a dependent source. In the presence of dependent source, R_{th} can be determined by finding V_{oc} & I_{sc}

$$\therefore R_{th} = \frac{V_{oc}}{i_{sc}}$$

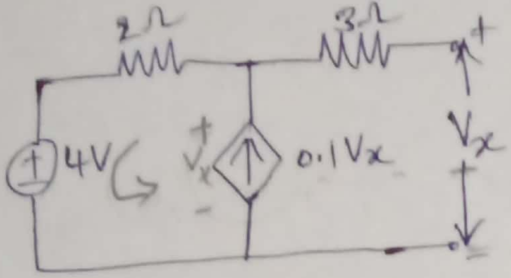


fig ⓐ

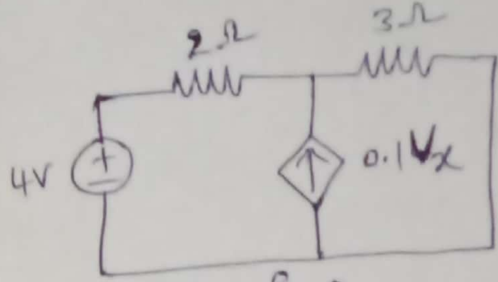


fig ⓑ

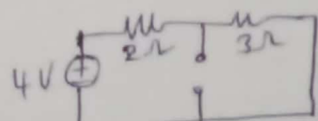
open ckt vtg can be found from the ckt shown in fig ⓐ. Since the o/p terminals are open, current passes through the 2Ω branch only.

$$V_x = 2 \times 0.1 V_x + 4$$

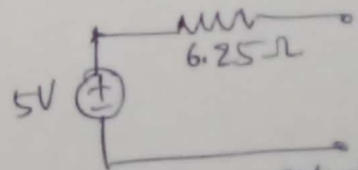
$$V_x = \frac{4}{0.8} = 5V$$

Short ckt current can be calculated from the ckt shown in fig ⓑ. Since $V_x = 0$, dependent current source is opened.

current $i_{sc} = \frac{4}{2+3} = 0.8A$



$$\therefore R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{5}{0.8} = 6.25\,\Omega$$



Thevenin's eqt ckt

$$-V_x + 2(0.1V_x) + 4 = 0$$

$$V_x = 0.2V_x + 4$$

Superposition Theorem:

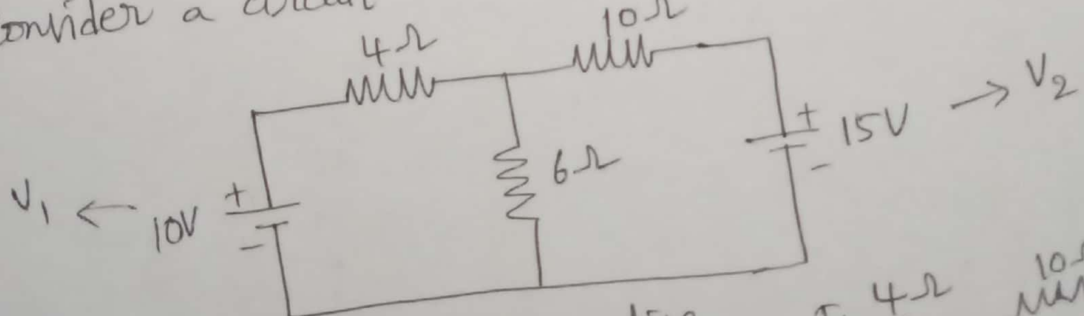
The principle of superposition helps us to analyze a linear ckt with more than one current or voltage sources sometimes it is easier to find out the voltage across (or) current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Statement: Any linear, bilateral two terminal network consisting of more than one sources, the total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually ~~wit~~ while other sources are replaced by their ideal internal resistances. (ie., voltage sources by a short ckt & current sources by open ckt).

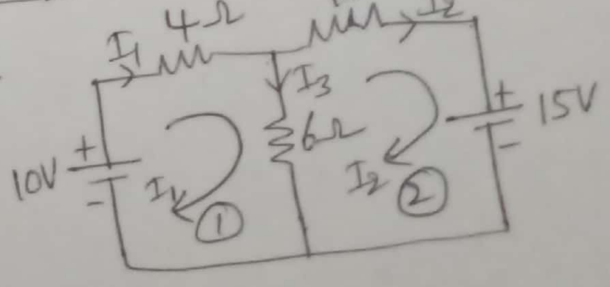
Steps to Apply superposition principle:

1. Replace all independent sources with their internal resistances except one source. Find the o/p (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other Independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Ex:- Consider a circuit



Sol:- Case 1: 10V and 15V acting



Apply KVL at loop 1

$$10 = 4I_1 + 6(I_1 - I_2)$$

$$10 = 10I_1 - 6I_2 \quad \text{— (1)}$$

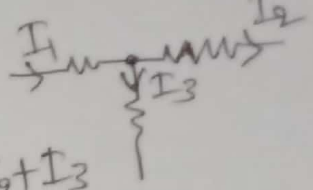
Apply KVL at loop ②

$$6(I_2 - I_1) + 10I_2 + 15 = 0$$

$$-6I_1 + 16I_2 = -15 \quad \text{—} ②$$

Solving the eqs ① & ②, $I_1 = 0.56\,A$

$I_2 = -0.72\,A$



$$\therefore I_3 = I_1 - I_2$$

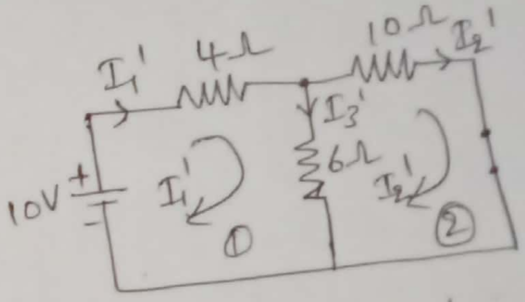
$$= 0.56 - (-0.72)$$

$$= 1.28\,A$$

$I_1 = I_2 + I_3$

$I_3 = I_1 - I_2$

Case ②: When only $V_1 = 10V$ is acting



Apply KVL to loop ①,

$$-10 + 4I_1' + 6(I_1' - I_2')$$

$$10I_1' - 6I_2' = 10 \quad \text{—} ③$$

$I_1' = I_2' + I_3'$

Apply KVL to loop ②,

$$6(I_2' - I_1') + 10I_2' = 0$$

$$-6I_1' + 16I_2' \quad \text{—} ④$$

Solving eqs ③ & ④ we get $I_1' = 1.3A$

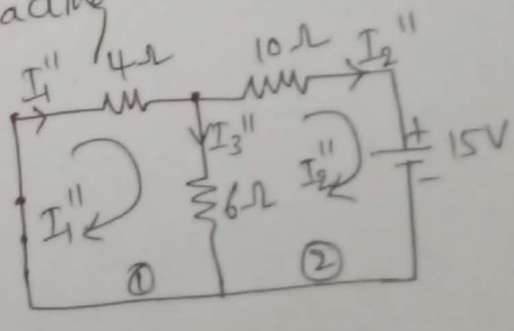
$I_2' = 0.48\,A$

1.30
0.48
0.82

$$\therefore I_3' = I_1' - I_2'$$

$$= 1.3 - 0.48 = 0.82\,A$$

Case ③: When only $V_2 = 15V$ source acting



Apply KVL to loop ①,

$$4I_1'' + 6(I_1'' - I_2'') = 0$$

$$10I_1'' - 6I_2'' = 0 \quad \text{—} ⑤$$

Apply KVL to loop ②,

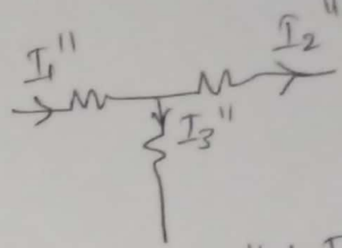
$$10I_2'' + 15 + 6(I_2'' - I_1'') = 0$$

$$16I_2'' - 6I_1'' = -15 \quad \text{—} ⑥$$

Solving the eqs ⑤ & ⑥ we get,

$I_1'' = -0.72\,A$

$I_2'' = -1.21\,A$



$I_1'' = I_2'' + I_3''$

$I_3'' = I_1'' - I_2''$

$\therefore I_3'' = I_1'' - I_2''$

$= -0.72 - (-1.21)$

$= 0.49\,A$

Applying superposition position principle,

$I_3 = I_3' + I_3''$

$1.28 = 0.49 + 0.82$

$\boxed{1.28 \approx 1.31}$

$I_1 = I_1' + I_1''$

$0.56 = 1.3 - 0.72$

$\boxed{0.56 \simeq 0.58}$

$I_2 = I_2' + I_2''$

$-0.72 = 0.48 - 1.2$

$\boxed{-0.72 \simeq -0.73}$

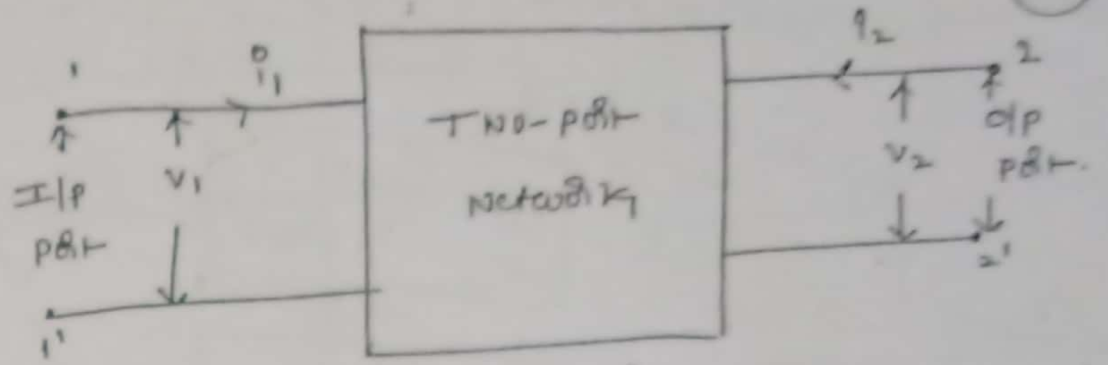
Hence Superposition theorem is verified for the given above ckt.

- Superposition principle is valid for only linear systems.
- It is not valid for power responses.
- It is applicable only for computing voltage & current responses.

TWO port Network & Network Theorem

(1) Z parameters or Impedance parameters or open-circuit parameters

(a)



A Two-port network having two ports, one is input port and second one output port. V_1 and V_2 are the input and o/p port voltages. I_1 and I_2 are the input and output port currents.

Function:- The Z parameters function is given by

$$\boxed{V = I\,[Z]} \quad -(1)$$

The Z-parameters equations are

$$V_1 = Z_{11} i_1 + Z_{12} i_2 \quad -(2)$$

$$V_2 = Z_{21} i_1 + Z_{22} i_2 \quad -(3)$$

The Matrix Form of the Z-parameters is given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad -(4)$$

Findout Z-parameters

Case-I Open the output port i.e $\boxed{i_2 = 0}$

$V_1 = Z_{11} i_1 + Z_{12} i_2$

$V_2 = Z_{21} i_1 + Z_{22} i_2$

$i_2 = 0$

$V_1 = Z_{11} i_1$ $\boxed{Z_{11} = \frac{V_1}{i_1} \Big|_{i_2 = 0}}$

$V_2 = Z_{21} i_1$ $\boxed{Z_{21} = \frac{V_2}{i_1} \Big|_{i_2 = 0}}$

Case-II open the i/p port i.e $\boxed{i_1 = 0}$

$V_1 = Z_{12} i_2$ $\boxed{Z_{12} = \frac{V_1}{i_2} \Big|_{i_1 = 0}}$

$V_2 = Z_{22} i_2$ $\boxed{Z_{22} = \frac{V_2}{i_2} \Big|_{i_1 = 0}}$

Z_{11} = Input port impedance

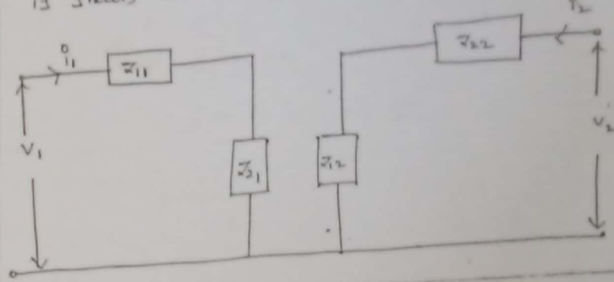
Z_{12} = Forword Transfer ~~o/p port~~ impedance

Z_{21} = Reverse Transfer impedance

Z_{22} = output port impedance.

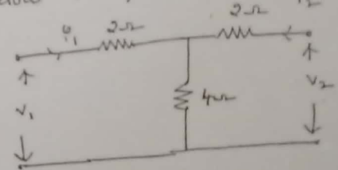
The impedance parameters are ... circuit parameters because the Z-parameters ... calculated by using open-circuit concept.

The equivalent circuit diagram of the Z-parameters is shown below figure.



Problems

(1) Find out Z or open ckt or Impedance parameters shown below the figure.

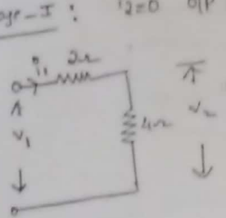


We know that Z-parameters

$V_1 = Z_{11} i_1 + Z_{12} i_2$

$V_2 = Z_{21} i_1 + Z_{22} i_2$

Case – I : $i_2=0$ o/p port is opnckt



$V_1 = i_1 \, R_{eq}$ $\quad R_{eq} = 2+4$

$R_{eq} = 6\Omega$

$\boxed{V_1 = 6 i_1}$

$V_2 = V_1 \times \frac{4}{4+2} = \frac{4}{6} V_1$

$V_2 = \frac{4}{\cancel{6}} \times \cancel{6} i_1 = 4 i_1$

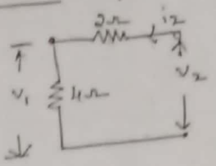
$\boxed{V_2 = 4 i_1}$

$i_2 = 0$ Hence

$Z_{11} = \frac{V_1}{i_1} = \frac{6\cancel{i_1}}{\cancel{i_1}} = 6\Omega$

$Z_{21} = \frac{V_2}{i_1} = \frac{4\cancel{i_1}}{\cancel{i_1}} = 4\Omega$

Case – II : $i_1=0$ i/p port is opnckt



$V_2 = i_2 \, R_{eq}$

$R_{eq} = 2+4 = 6\Omega$

$\boxed{V_2 = 6 i_2}$

$V_1 = V_2 \times \frac{4}{2+4}$

$V_1 = \frac{4}{6} V_2$

$V_1 = \frac{4}{\cancel{6}} \times \cancel{6} i_2$

$\boxed{V_1 = 4 i_2}$

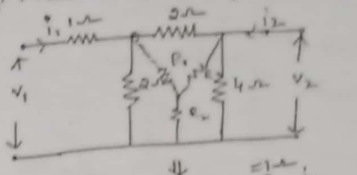
$Z_{12} = \frac{V_1}{i_2} = \frac{4\cancel{i_2}}{\cancel{i_2}} = 4\Omega$

$Z_{22} = \frac{V_2}{i_2} = \frac{6\cancel{i_2}}{\cancel{i_2}} = 6\Omega$

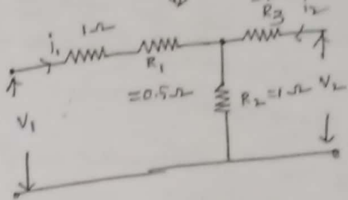
Finally

$Z_{11} = 6\Omega$	$Z_{12} = 4\Omega$
$Z_{21} = 4\Omega$	$Z_{22} = 6\Omega$

(2) Find the Z- parameters as shown below



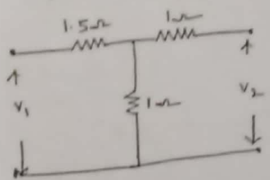
⇓



$R_1 = \frac{2\times2}{2+2+4} = \frac{4}{8} = 0.5\Omega$

$R_2 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$

$R_3 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$

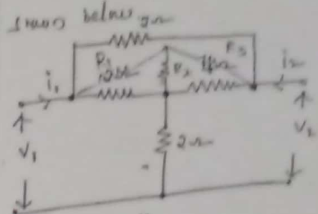


Note follow procedure same as problem (1) Verify the Answer

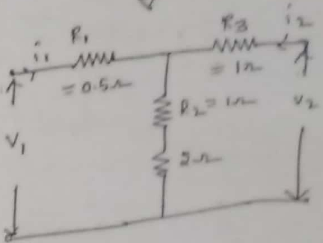
Ans:

$$Z = \begin{bmatrix} 2.5\Omega & 1\Omega \\ 1\Omega & 2\Omega \end{bmatrix}$$

(3) Find the Z - parameters as shown below



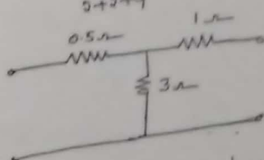
⇓



$R_1 = \frac{2\times2}{2+2+4} = \frac{4}{8} = \frac{1}{2} = 0$

$R_2 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$

$R_3 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\Omega$

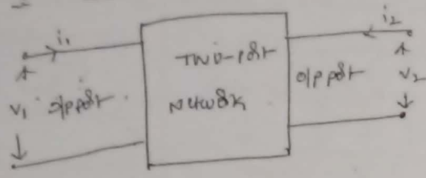


Note Follow procedure same as problem (1)

Verify the Answer

$$Z = \begin{bmatrix} Z_{11} = 3.5\Omega & Z_{12} = 3\Omega \\ Z_{21} = 3\Omega & Z_{22} = 4\Omega \end{bmatrix}$$

(2) Y or Admittance or Short-circuit Parameters



A two-port Network having two ports i.e i/p and output ports. V_1 and V_2 are the i/p and o/p port voltages. i_1 and i_2 are the i/p and o/p port currents.

The function of the Y-parameters is given by

$I = V[Y]$ —(1)

$i_1 = Y_{11}V_1 + Y_{12}V_2$ —(2) $\quad i_2 = Y_{21}V_1 + Y_{22}V_2$ —(3)

write down the matrix form of Y-parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad -(4)$$

Finding Y-parameters

$i_1 = Y_{11}V_1 + Y_{12}V_2$

$i_2 = Y_{21}V_1 + Y_{22}V_2$

Case-I $V_2 = 0$

o/p port is short ckt

$i_1 = Y_{11}V_1$

$$\boxed{Y_{11} = \frac{i_1}{V_1}\Big|_{V_2=0}}$$

$i_2 = Y_{21}V_1$

$$\boxed{Y_{21} = \frac{i_2}{V_1}\Big|_{V_2=0}}$$

Case-II $V_1 = 0$, i/p port is short circuit

$i_1 = V_2 Y_{12}$

$$\boxed{Y_{12} = \frac{i_1}{V_2}\Big|_{V_1=0}}$$

$i_2 = V_2 Y_{22}$

$$\boxed{Y_{22} = \frac{i_2}{V_2}\Big|_{V_1=0}}$$

Here

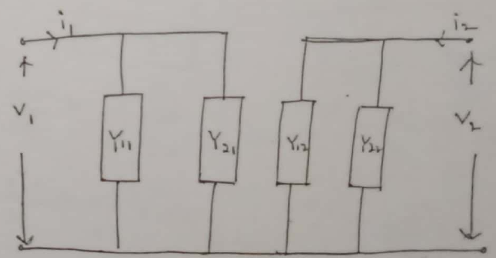
Y_{11} = i/p port Admittance

Y_{12} = Transfer Forward Admittance

Y_{21} = Transfer Reverse Admittance

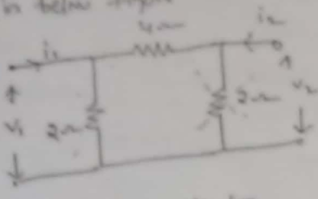
Y_{22} = o/p port Admittance.

The equivalent ckt of the Y-parameters are shown below.



The Y-parameters are also called short-circuit parameters because the Y-parameters are calculated by using short-circuit concept.

Q: Find out Y-parameters shown in below figure



we know that

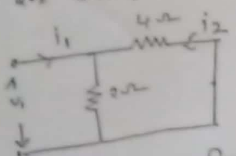
Y-parameters

$$\left.\begin{aligned} i_1 &= Y_{11}V_1 + Y_{12}V_2 \\ i_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}\right] \text{—} (1)$$

Case-I $V_2 = 0$

o/p port is short-ckt

$V_2 = 0$ means voltage across 2Ω resistor is zero.



$V_1 = i_1 R_{eq}$ $R_{eq} = \frac{2\times4}{2+4}$

$R_{eq} = \frac{8}{6}\,\Omega$

$\boxed{V_1 = \frac{8}{6} i_1}$

$i_2 = -i_1 \times \frac{2}{2+4} = \frac{-2i_1}{6}$

ioz

$Y_{11} = \frac{i_1}{V_1} = \frac{\frac{\cancel{6}}{\cancel{6}}\cancel{i_1}}{\frac{8}{6}\cancel{i_1}}$

$\boxed{Y_{11} = +\frac{6}{8}\,\mho}$

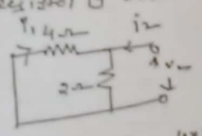
$Y_{21} = \frac{i_2}{V_1} = \frac{\frac{-2}{\cancel{6}}}{\frac{8}{\cancel{6}}}$

$\boxed{Y_{21} = -\frac{1}{4}\,\mho}$

Case-II $V_1 = 0$

I/p port is ~~open~~ short ckt

$V_1 = 0$ means voltage across 2Ω resistor is zero.



$V_2 = i_2 R_{eq}$ $R_{eq} = \frac{4\times2}{4+2} = 8/6\,\Omega$

$\boxed{V_2 = \frac{6}{8} i_2}$ $i_1 = -i_2 \times \frac{2}{2+4}$

$i_1 = -\frac{2}{6} i_2$

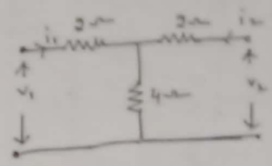
$Y_{12} = \frac{i_1}{V_2}$

$Y_{12} = \frac{-\frac{2}{\cancel{6}}\cancel{i_2}}{\frac{8}{\cancel{6}}\cancel{i_2}} = -\frac{1}{4}\,\mho$

$Y_{22} = \frac{i_2}{V_2} = \frac{\frac{6}{8}\cancel{i_2}}{\cancel{i_2}}$ $\frac{i_2}{\frac{6}{8}i_2}$

$Y_{22} = 8/6\,\mho$

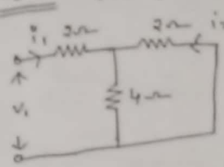
(2) Find out Y-parameters



$i_1 = Y_{11}V_1 + Y_{12}V_2$

$i_2 = Y_{21}V_1 + Y_{22}V_2$

Case-I $V_2 = 0$ o/p port is short ckt



$V_1 = R_{eq} i_1$ $R_{eq} = (2 \| 4) + 2$

$R_{eq} = \frac{2\times4}{2+4} + 2 = \frac{8}{6} + 2 =$

$R_{eq} = \frac{12+8}{6} = 20/6\,\Omega$

$\boxed{V_1 = \frac{20}{6} i_1}$

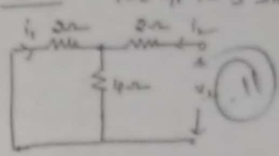
$i_2 = -i_1 \times \frac{4}{4+2} = \frac{-4}{6} i_1$

$\boxed{i_2 = -\frac{4}{6} i_1}$

$Y_{11} = \frac{i_1}{V_1} = \frac{\cancel{i_1}}{\frac{20}{6}\cancel{i_1}}$ $\boxed{Y_{11} = \frac{6}{20}\,\mho}$

$Y_{21} = \frac{i_2}{i_1} = \frac{-\frac{4}{6}\cancel{i_1}}{} = \boxed{\frac{-4}{6}\,\mho}$

Case-II $V_1 = 0$ I/p port is short



$V_2 = i_2 R_{eq}$ $R_{eq} = \frac{2\times4}{2+4} + 2$

$R_{eq} = \frac{8}{6} + 2 = \frac{20}{6}\,\Omega$

$\boxed{V_2 = \frac{20}{6} i_2}$

$i_1 = -i_2 \times \frac{4}{4+2} = \frac{-4}{6} i_2$

$\boxed{i_1 = -\frac{4}{6} i_2}$

$Y_{12} = \frac{i_1}{V_2} = \frac{-\frac{4}{\cancel{6}}\cancel{i_2}}{\frac{20}{\cancel{6}}\cancel{i_2}}$

$\boxed{Y_{12} = -\frac{1}{5}\,\mho}$

$Y_{22} = \frac{i_2}{V_2} = \frac{\cancel{i_2}}{\frac{20}{6}\cancel{i_2}}$

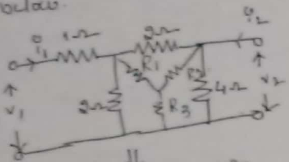
$Y_{22} = \frac{6}{20}\,\mho$

Finally

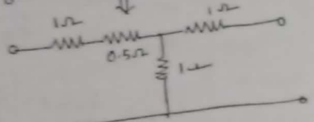
$Y_{11} = 6/20\,\mho$ $Y_{12} = -1/5\,\mho$

$Y_{21} = -4/6\,\mho$ $Y_{22} = \frac{-4}{6}\,\mho$

(3) Find out Y-parameters as shown below.



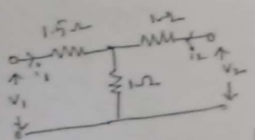
⇓



$$R_1 = \frac{2\times2}{2+2+4} = \frac{4}{8} = \frac{1}{2} = 0.5\,\Omega$$

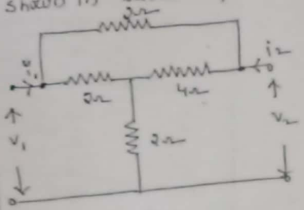
$$R_2 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\,\Omega$$

$$R_3 = \frac{2\times4}{2+2+4} = \frac{8}{8} = 1\,\Omega$$

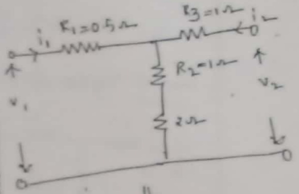


Note: Solve the above problem by refere problem number (2) in Y-parameters concept

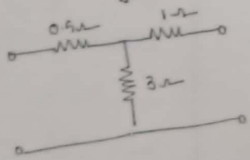
(4) Find out Z-parameters as shown in below figure



⇓

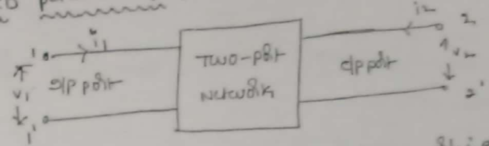


⇓



Note: Solve the above problem by refere problem number (2) in Y-parameters

ABCD parameters :-



A two-port network having two port i.e i/p and output ports. V_1 and V_2 are the input and output port voltages and i_1 and i_2 are the i/p and o/p port currents.

The ABCD parameters are

$$V_1 = AV_2 - Bi_2 \quad -(1)$$

$$i_1 = CV_2 - Di_2 \quad -(2)$$

The matrix form of ABCD-parameters

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix} \quad -(3)$$

Find out ABCD-parameters

$V_1 = AV_2 - Bi_2$ and $i_1 = CV_2 - Di_2 \quad -(4)$

Case-I $i_2 = 0$ output port is open circuit

$V_1 = AV_2$ $\quad$ $i_1 = CV_2$

$$\boxed{A = \frac{V_1}{V_2}\Big|_{i_2=0}} \qquad \boxed{C = i_1/V_2\Big|_{i_2=0}}$$

Case-II $V_2 = 0$ output port is short circuit

$V_1 = -Bi_2$ and $i_1 = -Di_2$ $\quad$ $D = -\frac{i_1}{i_2}\Big|_{V_2=0}$

$B = -V_1/i_2\Big|_{V_2=0}$

* ABCD parameters are also called Transmission parameters.

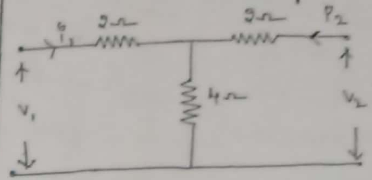
* The main difference between ABCD, Z and Y parameters are

(i) Z-parameters point of view the Z-parameters are calculated by using open-circuit concept

(ii) Y-parameters point of view the Y-parameters are calculated by using short-circuit concept

(iii) But ABCD parameters point of view the ABCD parameters calculated by using open-circuit and short-circuit concepts but referred to only ~~output~~ output port side only.

(1) Find out ABCD parameters as shown in below figure.

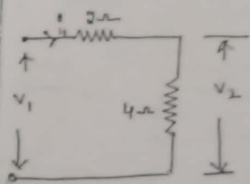


We know that ABCD parameters are

$V_1 = AV_2 - Bi_2$ —①

$i_1 = CV_2 - Di_2$ —②

Case-I Open the output port i.e $i_2 = 0$



$V_1 = i_1 R_{eq}$ $R_{eq} = 2+4 = 6\Omega$

$\boxed{V_1 = 6i_1}$ $\boxed{V_2 = 4i_1}$ $i_1 = \frac{V_2}{4}$

$V_1 = AV_2$ $A = \frac{V_1}{V_2} = \frac{6\cancel{i_1}}{4\cancel{i_1}}$

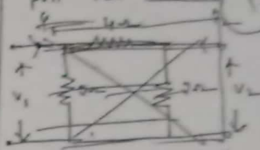
$\boxed{A = \frac{6}{4}}$

$C = \frac{i_1}{V_2} = \frac{\cancel{V_2}}{4\cancel{V_2}} = \frac{1}{4}$

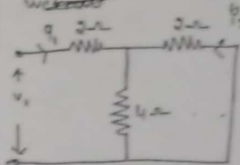
$\boxed{C = Y_4}$

Case-II Short the output port i.e $\boxed{V_2 = 0}$

12



~~we~~



$V_1 = i_1 R_{eq}$

$R_{eq} = 2 \| 4 + 2$

$= \frac{2 \times 4}{2+4} + 2 = \frac{8}{6}$

$\boxed{R_{eq} = 20/6\,\Omega}$ $\boxed{V_1 = \frac{20}{6}}$

$i_2 = -i_1 \times \frac{4}{4+2} = \frac{-4}{6}$

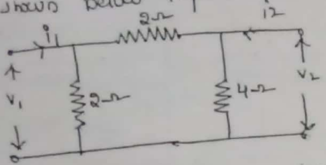
$\boxed{i_2 = -\frac{4}{6} i_1}$

$V_1 = -Bi_2$ $B = -\frac{V_1}{i_2}$

$B = -\frac{\frac{20}{6}\cancel{i_1}}{\frac{-4}{6}\cancel{i_1}} = 5$ $\boxed{B=}$

$D = -\frac{i_1}{i_2} = +\frac{\cancel{i_1}}{\frac{+4}{6}\cancel{i_1}} = \frac{6}{4}$ [

* Findout ABCD parameters as shown below figure

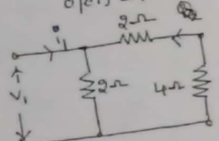


We know that ABCD parameters

$V_1 = AV_2 - Bi_2$ — (1)

$i_1 = CV_2 - Di_2$ — (2)

Case-I $\boxed{i_2=0}$ output port is open circuit



$V_1 = R_{eq} i_1$ $\quad R_{eq} = (4+2) \| 2$

$R_{eq} = \frac{6 \times 2}{6+2} = \frac{12}{8}\,\Omega$ $\quad \boxed{R_{eq} = \frac{12}{8}\,\Omega}$

$\boxed{V_1 = \frac{12}{8} i_1}$

$i_2 = -i_1 \times \frac{2}{2+2+4} = -\frac{2}{8} i_1$

$\boxed{i_2 = -\frac{2}{8} i_1}$

$V_1 = AV_2$ $\quad A = \frac{V_1}{V_2} = \frac{\frac{12}{8}\cancel{i_1}}{-\cancel{i_1}} = \frac{12}{8}$ $\quad \boxed{A = \frac{12}{8}}$

$V_2 = 4i_2$

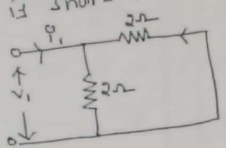
$V_2 = 4 \times -\frac{2}{8} i_1 = -\frac{\cancel{8}}{\cancel{8}} i_1 = -i_1$

$\boxed{V_2 = -i_1}$

$C = \frac{i_1}{V_2} = \frac{\cancel{i_1}}{-\cancel{i_1}} = -1$

$\boxed{C=-1}$

Case-II $\boxed{V_2=0}$ output port is short circuit



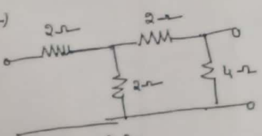
Note Easy method is

First convert total

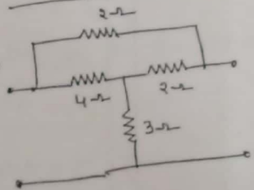
Π network into T-network

* practice the following networks

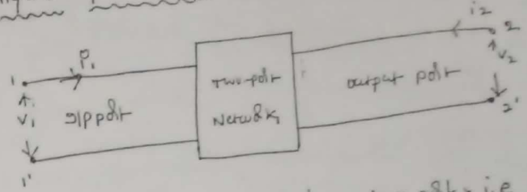
(1)



(2)



* Hybrid parameters :-



A Two-port network having two ports i.e Input and output ports. V_1 and V_2 are the input and output voltages. I_1 and I_2 are the input and output currents.

The Hybrid parameters are

$V_1 = h_{11} i_1 + h_{12} V_2$ — (1)

$i_2 = h_{21} i_1 + h_{22} V_2$ — (2)

The matrix Form of H-parameters are

$$\begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix} \quad \text{— (3)}$$

Findout H-parameters

$V_1 = h_{11} i_1 + h_{12} V_2$ and $i_2 = h_{21} i_1 + h_{22} V_2$

Case-I $V_2=0$ Short the o/p port terminals

$\boxed{h_{11} = \frac{V_1}{i_1}\Big|_{V_2=0}}$ and $\boxed{h_{21} = \frac{i_2}{i_1}\Big|_{V_2=0}}$

Case-II $i_1=0$ o/p port is open circuit

$V_1 = h_{12} V_2$ $\quad \boxed{h_{12} = V_1/V_2\Big|_{i_1=0}}$ $\quad i_2 = h_{22} V_2$ $\quad \boxed{h_{22} = \frac{i_2}{V_2}\Big|_{i_1=0}}$

* The main differnce b/w H, Z and Y parameters are

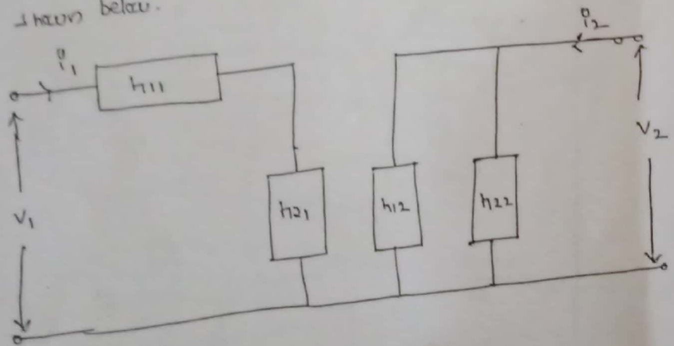
(i) Z-parameters are calculated by using open circuit concept

(ii) Y-parameters are calculated by using short circuit concept

(iii) ABCD parameters are calculated by using open and short circuit concepts but only output port side

(iv) The H-parameters are calculated by using input port is open circuit and output port is short circuit

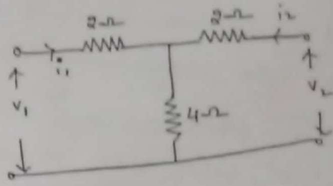
The equivalent circuit of the H-parameters are shown below.



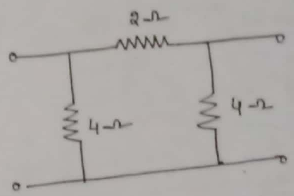
* Findout the H-parameters as shown in below figures.

13

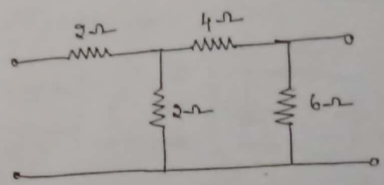
(1)



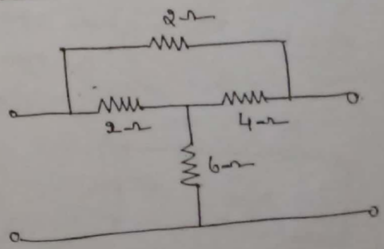
(2)



(3)



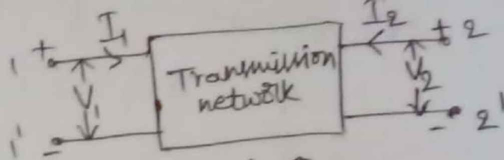
(4)



→ Transmission (or) ABCD parameters:-

- ABCD parameters are widely used in transmission line theory & cascade networks (→ ex: design of telephone sys, microwave n/ws, radars etc)
- Transmission parameters provide a direct relationship between i/p and o/p. These are also called as general ckt parameters (or) chain parameters.
- In describing the transmission parameters, the i/p variables V_1 & I_1 at port 1-1', usually called the Sending end, are expressed in terms of the o/p variables V_2 & I_2 at port 2-2', called the Receiving end. They are defined by,

$$V_1 = AV_2 - BI_2 \quad —①$$
$$I_1 = CV_2 - DI_2 \quad —②$$



fig①

The '-ve' sign is used with I_2, and not for the parameter B and D. Both port currents I_1 and '$-I_2$' are directed to the right, ie., with a -ve sign in eqs ① & ② the current at port 2-2' which leaves the port is designated as positive. The parameters A, B, C & D are called the transmission parameters.

In the matrix form, eqs ① & ② are expressed as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the "transmission matrix".

Case①: With port 2-2' open ie., $I_2 = 0$

Applying a vtg V_1 at the port 1-1',

$V_1 = AV_2 - 0 \Rightarrow \boxed{A = \frac{V_1}{V_2}\Big|_{I_2=0}}$ $\quad \frac{1}{A} = \frac{V_2}{V_1}\Big|_{I_2=0} = g_{21}\Big|_{I_2=0}$

$I_1 = CV_2 - 0 \Rightarrow \boxed{C = \frac{I_1}{V_2}\Big|_{I_2=0}}$ $\quad \frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21}$

$\frac{1}{A}$ = open ckt vtg gain, a dimensionless parameter

$\frac{1}{C} = Z_{21}$ = open ckt transfer impedance.

Case②:- With port 2-2' short ckted ie., $V_2 = 0$

Applying vtg V_1 at port 1-1', $\quad V_1 = 0 - BI_2$

$I_1 = 0 - DI_2$

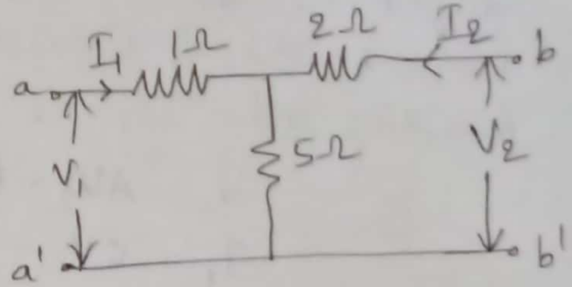
$$V_1 = 0 - BI_2 \Rightarrow \boxed{-B = \frac{V_1}{I_2}\Big|_{V_2=0}}$$

$$I_1 = 0 - DI_2 \Rightarrow \boxed{-D = \frac{I_1}{I_2}\Big|_{V_2=0}}$$

$$-\frac{1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0} = Y_{21} = \text{short ckt transfer admittance}$$

$$-\frac{1}{D} = \frac{I_2}{I_1}\Big|_{V_2=0} = \alpha_{21}\Big|_{V_2=0} = \text{short ckt current gain} = \text{dimensionless parameter.}$$

Ex ① Find the transmission or general ckt parameters for the ckt shown.



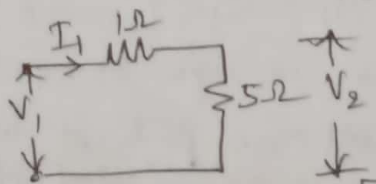
Sol:-

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

When b-b' is open, $I_2 = 0$, $A = \frac{V_1}{V_2}\Big|_{I_2=0}$

where $\boxed{V_1 = 6I_1}$



$R_{eqt} = 1+5 = 6\Omega$

$V_1 = I_1 R_{eqt} = I_1(6)$

$V_2 = V_1 \times \frac{5}{(1+5)} = \frac{5}{6}V_1$

$$V_2 = V_1 \times \frac{5}{(1+5)}$$

$$= \frac{5}{6}V_1 = \frac{5}{6}(6I_1) = 5I_1 \Rightarrow \boxed{V_2 = 5I_1}$$

$$\therefore A = \frac{V_1}{V_2} = \frac{6I_1}{5I_1} = \frac{6}{5}$$

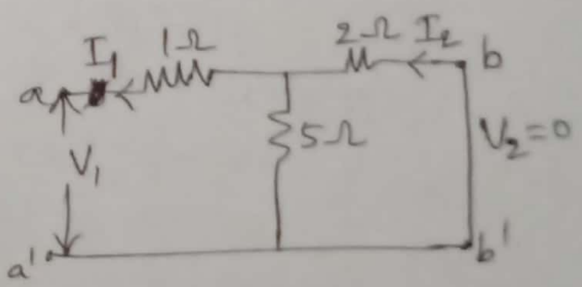
$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{1}{5}\ \mho$$

$V_2 = 5I_1$

$I_1 = \frac{V_2}{5}$

$$C = \frac{I_1}{V_2} = \frac{(V_2/5)}{V_2} = \frac{V_2}{5V_2} = \frac{1}{5}$$

When b-b' is short ckted, $V_2 = 0$



$$B = \frac{-V_1}{I_2}\Big|_{V_2=0}$$

$$-I_2 = \frac{5}{17}V_1 \Rightarrow B = \frac{17}{5}\Omega$$

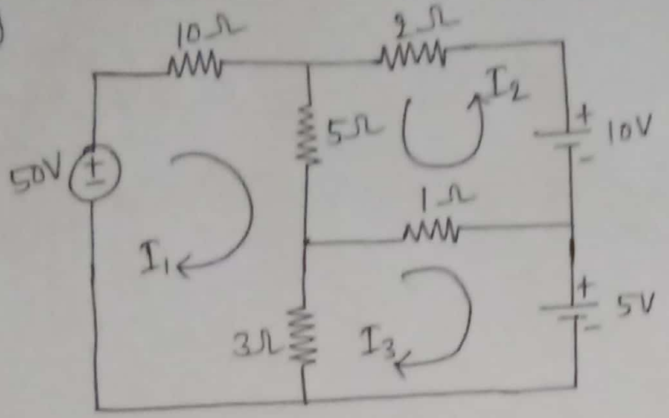
$$D = \frac{-I_1}{I_2}\Big|_{V_2=0}$$

|||ly $I_1 = \frac{7}{17}V_1$ and $-I_2 = \frac{5}{17}V_1$

$$D = \frac{7}{5}$$

Mesh Analysis:

(1)



Determine the mesh currents I_1, I_2, I_3 for the ckt shown.

same direction → +
opposite direction → −

Sol:-

$$-50 + 10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 0$$
$$-50 + 18I_1 + 5I_2 - 3I_3 = 0$$
$$18I_1 + 5I_2 - 3I_3 = 50 \quad \text{——} \ ①$$

$$-10 + 2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 0$$
$$8I_2 + 5I_1 + I_3 = 10$$
$$5I_1 + 8I_2 + I_3 = 10 \quad \text{——} \ ②$$

$$5 + 3(I_3 - I_1) + 1(I_3 + I_2) = 0$$
$$5 + 4I_3 - 3I_1 + I_2 = 0$$
$$4I_3 - 3I_1 + I_2 = -5$$
$$-3I_1 + I_2 + 4I_3 = -5 \quad \text{——} \ ③$$

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix} = 18(32-1) - 5(20+3) - 3(5+24)$$
$$18(31) - 5(23) - 3(29)$$
$$558 - 115 - 87$$
$$= 356$$

$$\Delta_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\Delta} = \frac{50(32-1) - 5(40+5) - 3(10+40)}{356} = \frac{1175}{356} = 3.3\text{A}$$

$$\therefore I_1 = 3.3\text{A}$$

$$\Delta_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\Delta} = \frac{18(40+5) - 50(20+3) - 3(-25+30)}{356}$$

$$= \frac{810 - 1150 - 15}{356} = \frac{-355}{356} = -0.997\ A$$

$\therefore I_2 = -0.997\ A$

$$\Delta_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\Delta} = \frac{18(-40-10) - 5(-25+30) + 50(5+24)}{356}$$

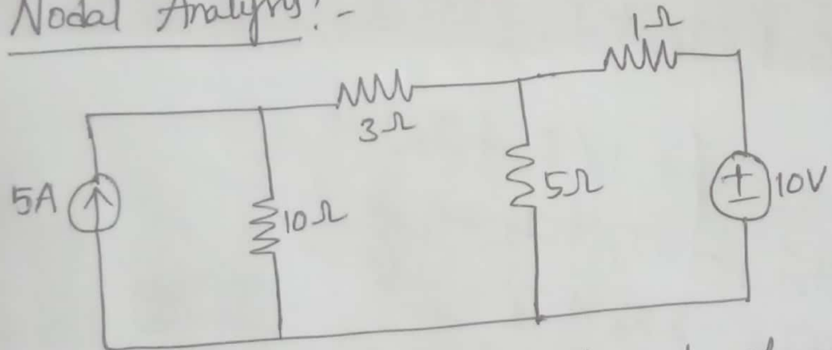
$$= \frac{-900 - 25 + 1450}{356} = \frac{525}{356} = 1.47\ A$$

$\therefore I_3 = 1.47\ A$

$\therefore I_1 = 3.3A$; $I_2 = -0.997A$; $I_3 = 1.47\ A$

→ Nodal Analysis :-

(1)



Write node voltage equations & determine the currents in each branch for the network shown.

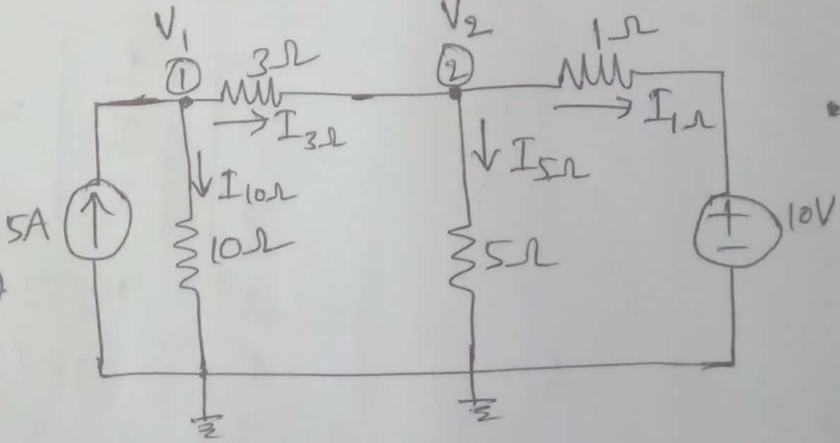
Sol:- Step①: Assign voltages at each node shown in fig①.

Applying KCL at node ①,

$$5 = \frac{V_1}{10} + \frac{V_1 - V_3}{3}$$

$$\Rightarrow V_1\left[\frac{1}{10} + \frac{1}{3}\right] - V_2\left[\frac{1}{3}\right] = 5$$

$$\frac{13}{30}V_1 - \frac{1}{3}V_2 = 5 \quad \text{—①}$$



fig①

Applying KCL at node ②,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$\frac{V_2-V_1}{3}+\frac{V_2}{5}+\frac{V_2-10}{1}=0$$

$$-V_1\left[\frac{1}{3}\right]+V_2\left[\frac{1}{3}+\frac{1}{5}+1\right]=10$$

$$-V_1\left[\frac{1}{3}\right]+V_2\left[\frac{23}{15}\right]=10 \quad \text{——} (2)$$

$$\frac{1}{3}+\frac{1}{5}+1 \qquad \frac{5+3+15}{15} \qquad \frac{23}{15}$$

Solving (1) & (2), $V_1 = 19.85\ V$

$V_2 = 10.9\ V$

$$\therefore\ I_{10\Omega}=\frac{V_1}{10}=\frac{19.85}{10}=1.985\ A$$

$$I_{3\Omega}=\frac{V_1-V_2}{3}=\frac{19.85-10.9}{3}=2.98\ A$$

$$I_{5\Omega}=\cancel{\frac{V_1-V_2}{3}}=\frac{V_2}{5}=\frac{10.9}{5}=2.18\ A$$

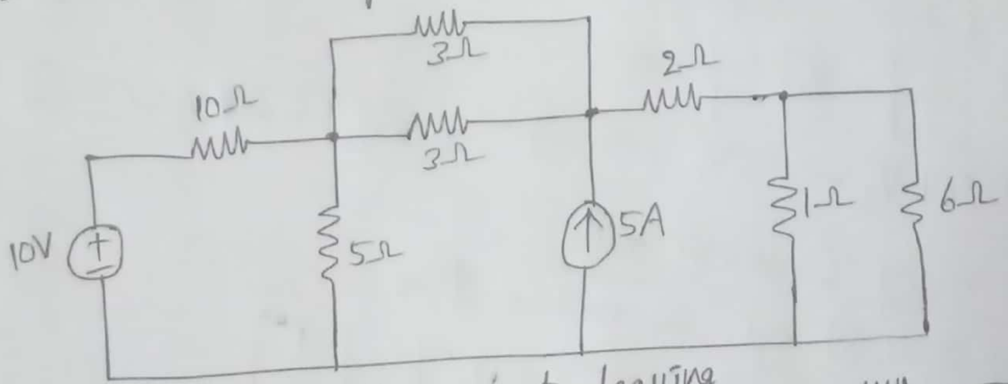
$$I_{1\Omega}=\frac{V_2-10}{1}=0.9\ A$$

(2) Determine the voltages at each node ~~for~~ the circuit shown.

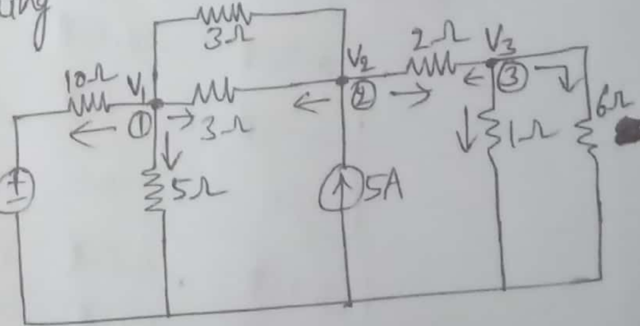
Sol:- $V_1 = 8.06V$

$V_2 = 10.2V$

$V_3 = 3.07V$



Sol:- At node (1), assuming all currents leaving apply KCL



$$\frac{V_1-10}{10}+\frac{V_1}{5}+\frac{V_1-V_2}{3}+\frac{V_1-V_2}{3}=0$$

At node (2), apply KCL,

$$\frac{V_2-V_1}{3}+\frac{V_2-V_1}{3}+\frac{V_2-V_3}{2}=5$$

At node (3), apply KCL

$$\frac{V_3-V_2}{2}+\frac{V_3}{1}+\frac{V_3}{6}=0$$

$$\frac{V_1 - 10}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{3} = 0$$

$$V_1\left[\frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{3}\right] - V_2\left[\frac{1}{3} + \frac{1}{3}\right] - \frac{10}{10} = 0$$

$$V_1\left[\frac{29}{30}\right] - V_2\left[\frac{2}{3}\right] = 1$$

$$0.967\,V_1 - 0.667\,V_2 = 1 \quad \text{—— } (1)$$

$$\frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{3}$$

$$\frac{3+6+10+10}{30}$$

$$\frac{29}{30}$$

3 | 10, 5, 3, 3
5 | 10, 5, 1, 1
2, 1, 1, 1

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2} - 5 = 0$$

$$V_2\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right] - V_1\left[\frac{1}{3} + \frac{1}{3}\right] - V_3\left[\frac{1}{2}\right] = 5$$

$$V_2\left[\frac{7}{6}\right] - V_1\left[\frac{2}{3}\right] - V_3\left[\frac{1}{2}\right] = 5$$

$$-0.667\,V_1 + 1.167\,V_2 - 0.5V_3 = 5 \quad \text{—— } (2)$$

$$\frac{2}{3} + \frac{1}{2} = \frac{4+3}{6}$$

2 | 2, 6
1, 3

$$\frac{V_3 - V_2}{2} + \frac{V_3}{1} + \frac{V_3}{6} = 0$$

$$V_3\left[\frac{1}{2} + 1 + \frac{1}{6}\right] - V_2\left[\frac{1}{2}\right] = 0$$

$$V_3\left[\frac{5}{3}\right] - V_2\left[\frac{1}{2}\right] = 0$$

$$1.667\,V_3 - 0.5V_2 = 0 \Rightarrow -0.5V_2 + 1.667\,V_3 = 0 \quad \text{—— } (3)$$

$$\frac{1}{2} + 1 + \frac{1}{6}$$

$$\frac{3+6+1}{6}$$

$$\frac{10}{6} = \frac{5}{3}$$

Applying Cramer's rule,

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix} \rightarrow \Delta} = \frac{7.154}{0.887} = 8.06\,V$$

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{0.887} = \frac{9.06}{0.887} = 10.2\,V$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{0.887} = \frac{2.73}{0.887} = 3.07\,V$$