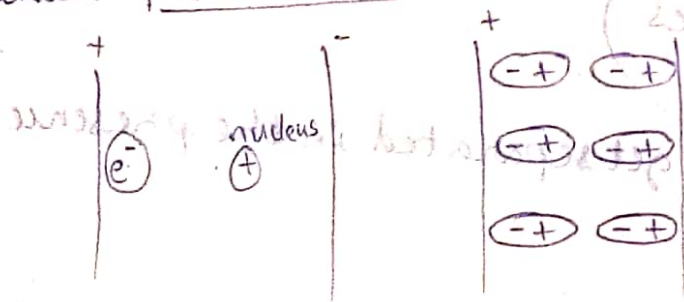


Dielectrics

Dielectrics are insulators - in which the electrons are tightly bounded to the nucleus and there are no free electrons are present for conductivity.

When these type of materials are placed in an electric field then separation of +ve and -ve charge takes place. This phenomenon is called polarisation and the material is called dielectric material.

Ex: Mica, Bakelite, oils, water, etc.



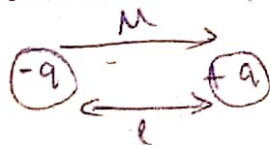
Electric dipole :-

Two opposite charges of ^{same} magnitude separated by a finite distance is called electric dipole.



Electric dipole moment :-

The product of one of the charge and charge separating distance is called electric dipole moment.



$$\mu = q \times l$$

Units

coulomb.metre = debye

$$1 \text{ debye} = 3.3 \times 10^{-30} \text{ C}\cdot\text{m}$$

Types of dielectrics

1) Polar dielectric :-

Molecules in which +ve and -ve charge does not coincide with each other and net dipole moment is called polar dielectric.

Ex :- H_2O , nitrobenzene

2) Non polar dielectric :-

Molecules in which +ve and -ve charge coincide with each other and net dipole moment does not exist is called non polar dielectric

Ex :- CO_2 , H_2 , N_2

⇒ Dielectric Polarisation :-

When a dielectric material is placed in an electric field, separation of negative and positive charge takes place. Each separated charges will act as the dipole and hence exhibit dipole moment. This phenomenon is called polarisation (P).

It is defined as ratio of electric dipole moment per unit volume

$$\boxed{P = \frac{M}{V}} = \frac{C \cdot m}{m^3} = C/m^2$$

Let N be the no. of atoms or molecules per unit volume ($V=1$) and μ be the net dipole moment then, $\boxed{P = N\mu}$ - ①

⇒ Dielectric Polarizability :-

(The ability of to separate the charges)
to allow to get

The ability to allow the charges to get separated in the presence of electric field.

$$\mu = \alpha E$$

$$\mu = \alpha E$$

$$\boxed{\alpha = \frac{\mu}{E}} \text{ } \frac{C \cdot m}{Fm^2} \text{ - ②}$$

It is defined as the ratio of electric dipole moment per unit electric field.

From ① & ②

$$\boxed{P = N\alpha E}$$

Electric Susceptibility

When an electric field is applied separation of +ve and -ve charges takes place and hence polarisation takes place.

$$P \propto E$$

$$P = \chi E$$

$$\boxed{\chi = \frac{P}{E}}$$

Permittivity :-

It represents easily polarisable nature of the dielectric material. Permittivity of dielectric material is denoted by ϵ and Permittivity of free space, $\boxed{\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}}$

Relative Permittivity (ϵ_r) (Dielectric constant (K)) :-

It is defined as the ratio of permittivity of a material to the free space.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$$

Dielectric Flux Density (D) :-

It is defined as product of permittivity of a material and applied electric field

$$\boxed{D = \epsilon E}$$

$$\Rightarrow \boxed{D = \epsilon_r \epsilon_0 E}$$

$$\Rightarrow \boxed{D = K \epsilon_0 E}$$

Relation b/w χ and ϵ_r

Total flux passing through the material is equal to flux through free space and polarisation.

$$D = D_0 + P$$

$$\epsilon E = \epsilon_0 E + P$$

$$\epsilon_r \epsilon_0 E - \epsilon_0 E = P$$

$$\Rightarrow \epsilon_0 E (\epsilon_r - 1) = P \Rightarrow \boxed{D_0 (\epsilon_r - 1) = P}$$

$$\Rightarrow \epsilon_0 (\epsilon_r - 1) = \frac{P}{E}$$

$$\Rightarrow \epsilon_0 (\epsilon_r - 1) = \chi$$

$$\Rightarrow \boxed{\chi = \epsilon_0 (\epsilon_r - 1)}$$

Magnetic material

$$\mu_m = m \times \epsilon l$$

$$= \Sigma \times A$$

$$M = \frac{\mu}{V}$$

$$\chi = \frac{M}{H}$$

$$B = \frac{\Phi}{A}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

Dielectric material

$$\mu = q \times l$$

$$P = \frac{\mu}{V}$$

$$\chi = \frac{P}{E}$$

$$D = \epsilon E$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Types of Polarisation

- 1) Electronic
- 2) Ionic
- 3) Orientational

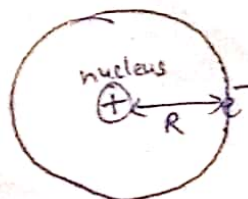
1) Electronic Polarisation

The displacement of +ve charge nucleus and -ve charge electrons in opposite direction by the application of electric field is called electronic polarisation.

Let us consider an atom \rightarrow nucleus and around which electrons are revolving in a circular orbit of radius R .

In the absence of electric field

Let Ze be the charge of the nucleus (protons)
 $-Ze$ be the charge of the electrons

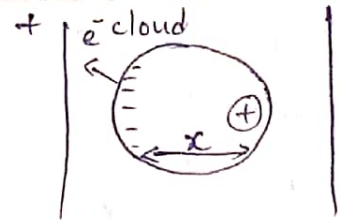


$$\text{Charge density} = \frac{\text{Charge}}{\text{Volume}}$$

$$R = \frac{-Ze}{\frac{4}{3}\pi R^3} \quad \text{--- (1)}$$

In the presence of electric field

Due to application of \vec{e} field let +ve charge nucleus and -ve charge e^- (e^- cloud) are separated by a distance x .



The Lorentz force acting b/w the two charges are Ze , $-Ze$

$$F_L = e \cdot E$$

$$F_L = -ZeE \quad \text{--- (2)}$$

Coulomb's force of attraction acting on it is

$$F_C = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$F_C = \frac{1}{4\pi\epsilon_0} \frac{(-Ze)(Ze)}{x^2} \quad \text{--- (3)}$$

charge density = $\frac{\text{charge}}{\text{volume}}$

$$\rho = \frac{-Ze}{\frac{4}{3}\pi x^3}$$

$$-Ze = \rho \times \frac{4}{3}\pi x^3 \quad \text{--- (4)}$$

substitute ' ρ ' from (4) in (3)

$$-Ze = \frac{-Ze}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi x^3$$

$$-Ze = \frac{-Ze}{R^3} x^3 \quad \text{--- (5)}$$

substitute (5) in (3)

$$F_C = \frac{1}{4\pi\epsilon_0} \frac{-Ze x^3 Ze}{R^3 x^2}$$

At equilibrium, both the forces (F_L, F_C) are equal

$$F_L = F_C$$

$$+ZeE = \frac{1}{4\pi\epsilon_0} \frac{Ze x^3 Ze}{R^3 x^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{x Ze}{R^3} \quad \Rightarrow \quad x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

In the presence of \vec{e} field separation of charges takes place each separated charge acts as dipole and exhibit dipole moment

$$\mu = q \times l$$

$$\mu = ze \times l$$

$$\mu = ze \times \frac{4\pi\epsilon_0 R^3 E}{ze}$$

$$\mu = 4\pi\epsilon_0 R^3 E$$

W.K.T,

$$\mu = \alpha E$$

$$\alpha e = 4\pi\epsilon_0 R^3$$

W.K.T,

$$P = NM$$

$$P = N 4\pi\epsilon_0 R^3 E \quad \text{--- (6)}$$

W.K.T

$$P = \epsilon_0 E (\epsilon_r - 1) \quad \text{--- (7)}$$

$$N 4\pi\epsilon_0 R^3 E = \epsilon_0 E (\epsilon_r - 1)$$

$$N 4\pi\epsilon_0 R^3 = \epsilon_0 (\epsilon_r - 1)$$

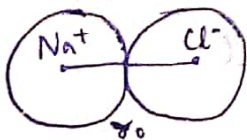
$$N \alpha e = \epsilon_0 (\epsilon_r - 1)$$

$$\alpha e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

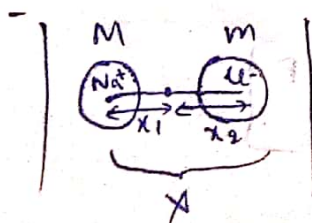
2) Ionic Polarisation

The displacement of cation and anion in opposite direction by the application of electric field is called ionic polarisation.

In the absence of \vec{e} field



In the presence of \vec{e} field



+ let M & m be the masses of cation & anion which are separated by a distance of x_1 & x_2 by the application of \vec{e} field.
Let ' e ' be the charge of the ions

This separated ions act as a dipole and hence there exists dipole moment

$$\mu = q \times l$$

$$= e \times X$$

$$\boxed{\mu = e(x_1 + x_2)} \quad - (1)$$

Lorentz force acting on the ions is $\boxed{F = eE}$

In addition to this another force called restoring force also acts which tries to make $x_1 = x_2 = 0$ (original position)

$$\boxed{F = Kx}$$

where $\boxed{K = M\omega^2}$

$$F = Kx_1$$

$$F = Kx_2$$

$$F = M\omega^2 x_1$$

$$F = m\omega^2 x_2$$

$$eE = M\omega^2 x_1$$

$$eE = m\omega^2 x_2$$

$$\boxed{x_1 = \frac{eE}{M\omega^2}} \quad \rightarrow \quad \boxed{x_2 = \frac{eE}{m\omega^2}} \quad - (2)$$

substitute (2) in (1),

$$\mu = e \left(\frac{eE}{M\omega^2} + \frac{eE}{m\omega^2} \right)$$

$$\mu = \frac{e^2 E}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right)$$

w.k.T, $\boxed{\alpha = \frac{\mu}{E}}$

$$\mu = \alpha E$$

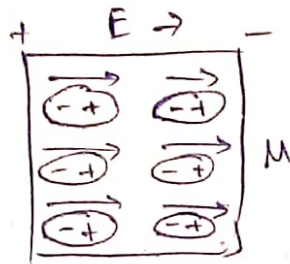
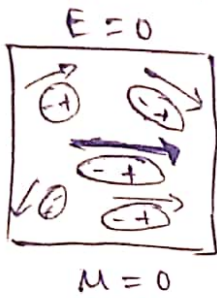
$$\boxed{\alpha_i = \frac{e^2}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right)}$$

↳ Orientalional Polarisation :-

This type of polarisation is observed only in polar dielectrics. In the absence of \vec{E} field all the dipole moments are oriented randomly and hence the net dipole moment is zero i.e., it is not polarised.

In the presence of \vec{E} field all the dipole moments are oriented in the same direction to the applied field, hence there

exist a dipole moment and it is said to be polarised. This type of polarisation is called orientational polarisation.



$$P = NM$$

$$P = \frac{NM^2}{3k_B T} E$$

w.k.T

$$P = N \alpha E$$

$$\alpha_0 = \frac{\mu^2}{3k_B T}$$

⇒ Total polarisability,

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

$$\alpha = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3k_B T}$$

⇒ Total polarisation,

$$P = N \alpha E$$

$$P = N \left[4\pi\epsilon_0 R^3 + \frac{e^2}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3k_B T} \right] E$$

Langevin Debye equation

* Q) Dielectric constant of He gas at NTP is 1.0000684 calculate the electronic polarisability of He atom with the gas contains 2.75×10^{25} atoms/m³

$$\epsilon_r = 1.0000684$$

$$N = 2.75 \times 10^{25}$$

$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

$$\epsilon_0 = 8.854 \times 10^{-12}$$

$$\Rightarrow \alpha_e = 2.20223 \times 10^{-41} \text{ Fm}^2$$

Q) A solid elemental dielectric has 3×10^{28} atoms/m³ shows an electronic polarisability 10^{-40} Fm² calculate dielectric constant

$$N = 3 \times 10^{28}$$

$$\alpha_e = 10^{-40}$$

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

$$(\epsilon_r - 1) = \frac{N \alpha_e}{\epsilon_0}$$

$$\epsilon_r = 1 + \frac{N \alpha_e}{\epsilon_0}$$

$$\boxed{\epsilon = 1.338}$$

Q) The relative permittivity of Sulphur is 4. Calculate its electronic polarisability, given that cubic sulphur has a density of $2.08 \times 10^3 \text{ kg/m}^3$ and its atomic weight is 32.

$$\epsilon_r = 4$$

$$\rho = 2.08 \times 10^3$$

$$\boxed{N = \frac{N_A \times \rho}{\text{At. weight}}}$$

$$\epsilon_r = 4$$

$$N = \frac{6.023 \times 10^{23} \times 2.08 \times 10^3}{32}$$

$$N = 3.914 \times 10^{25} \text{ atoms/m}^3$$

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

$$= \frac{8.854 \times 10^{-12} (3)}{3.914 \times 10^{25}}$$

$$\boxed{\alpha_e = 6.786 \times 10^{-37}}$$

Q) A parallel plate capacitor having area $6.45 \times 10^4 \text{ m}^2$ and ~~plate~~ a plate separation of $2 \times 10^{-3} \text{ m}$, across which a potential 12 V is supplied for a material having dielectric constant 5. Compute the polarisation.

$$A = 6.45 \times 10^4 \text{ m}^2$$

$$d = 2 \times 10^{-3} \text{ m}$$

$$V = 12, \epsilon_r = 5$$

$$E = \frac{V}{d}$$

$$E = 6000$$

$$P = \epsilon_0 E (\epsilon_r - 1)$$

$$P = 8.854 \times 10^{-12} \times 6 \times 10^3 \times 4$$

$$P = 2.12496 \times 10^{-7} \text{ C/m}^2$$

a) If radius of hydrogen atom is 0.055 nm then calculate electronic polarisability and relative permittivity. Given no. of atoms in 'H' gas is 9.8×10^{26} atoms/m³

Sol $N = 9.8 \times 10^{26}$ atoms/m³

$$R = 0.055 \times 10^{-9} \text{ m}$$

$$\alpha_e = 4\pi\epsilon_0 R^3$$

$$\alpha_e = 1.8511 \times 10^{-41} \text{ Fm}^2$$

$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

$$\epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0}$$

$$\epsilon_r = 1.002$$

b) If the permittivity of dielectric material 2.40×10^{-10} then calculate its dielectric constant and susceptibility.

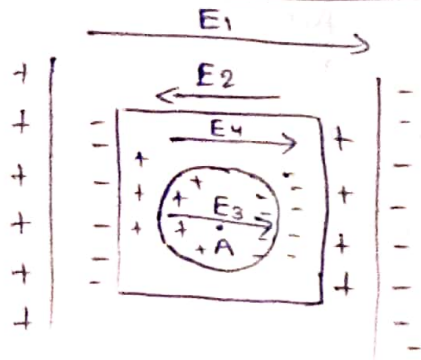
$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\epsilon_r = \frac{2.40 \times 10^{-10}}{8.854 \times 10^{-12}} = 27.10$$

$$\chi = \epsilon_0 (\epsilon_r - 1)$$

$$\chi = 2.31 \times 10^{-10}$$

Local field (or) Internal field (or) Lorentz field



The total resultant electric field acting on the atom present inside the polarised dielectric medium is called local field.

(The total electric field acting on the atom consist of)

let us consider a parallel plate capacitor in which a dielectric material is placed. Now, the dielectric material is polarised in the field across the plates.

The total electric field acting on the atom consist of 4 components

$$E_i = E_1 + E_2 + E_3 + E_4 \quad \text{--- (1)}$$

- E_1 - Electric field due to charged capacitor
- E_2 - Electric field due to induced charges in the dielectric material
- E_3 - Field due to atoms (dipole) present inside spherical region
- E_4 - Field due to surface charges in the spherical region

E_1

$$D = D_0 + P$$

$$\epsilon_0 E_1 = \epsilon_0 E + P$$

$$E_1 = E + \frac{P}{\epsilon_0}$$

E_2

$$D = D_0 + P$$

$$\epsilon_0 E_2 = 0 + P$$

$$E_2 = \frac{P}{\epsilon_0}$$

E_3 for cubic structure - symmetrical distribution of charges

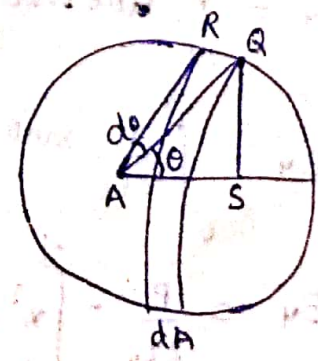
Hence, $E_3 = 0$

E_4 let us consider a small, area dA which is lying between θ & $\theta + d\theta$

$$A = \pi r^2$$

$$dA = 2\pi r$$

$$dA = 2\pi (SQ)(RQ)$$



To find sq

Consider $\triangle ASQ$,

$$\sin \theta = \frac{SQ}{AQ}$$

$$\sin \theta = \frac{SQ}{r}$$

$$SQ = r \sin \theta$$

$$\therefore dA = 2\pi r^2 \sin \theta d\theta$$

w.k.t,

$$P = \frac{M}{V}$$

$$P = \frac{q \times l}{l \times b \times h} = \frac{q \times l}{l \times A} = \frac{q}{A}$$

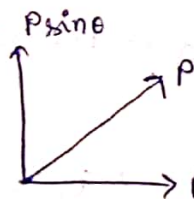
$$\Rightarrow P = \frac{q}{A}$$

$$q = PA$$

$$dq = P dA$$

$$dq = P \cos \theta dA$$

$$dq = P \cos \theta (2\pi r^2 \sin \theta d\theta)$$



(Polarisation takes place only in this direction)

The field due to charge dq on the atom at A is

$$dE_y = \frac{dq \cos \theta}{4\pi \epsilon_0 r^2}$$

$$dE_y = \frac{P}{4\pi \epsilon_0 r^2} 2\pi r^2 \sin \theta \cos^2 \theta d\theta$$

$$dE_y = \frac{P}{2\epsilon_0} \sin \theta \cos^2 \theta d\theta$$

$$\int dE_y = \int_0^\pi \frac{P}{2\epsilon_0} \sin \theta \cos^2 \theta d\theta$$

$$E_y = \frac{P}{2\epsilon_0} \int_1^{-1} x^2 dx$$

Let $\cos \theta = x$
 $-\sin \theta d\theta = dx$

$$E = \frac{F}{q}$$

$$E = \frac{1}{4\pi \epsilon_0} \frac{q^2}{r^2}$$

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

$$E_4 = \frac{P}{2\epsilon_0} \int_{-1}^1 x^2 dx$$

$$= \frac{P}{2\epsilon_0} \left[\frac{x^3}{3} \right]_{-1}^1$$

$$= \frac{P}{2\epsilon_0} \left(\frac{1}{3} + \frac{1}{3} \right)$$

$$= \frac{P}{2\epsilon_0} \left(\frac{2}{3} \right)$$

$$\boxed{E_4 = \frac{P}{3\epsilon_0}}$$

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$= E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

$$\boxed{E_i = E + \frac{P}{3\epsilon_0}}$$

→ Clausius - Mosotti equation

The relation between dielectric constant and polarisability of a dielectric material is called Clausius - Mosotti equation.

Let us consider a polarised dielectric material in which local field is E_i

$$\mu \propto E_i$$

$$\mu = \alpha E_i$$

$$\text{Polarisability} \rightarrow \boxed{\alpha = \frac{\mu}{E_i}}$$

If N is the no. of atoms per unit volume and μ is the average dipole moment then polarisation is given by

$$P = N\mu$$

$$\boxed{P = N\alpha E_i}$$

$$P = Nd \left(E + \frac{P}{3\epsilon_0} \right)$$

$$P = NdE + \frac{NdP}{3\epsilon_0}$$

$$P \left(1 - \frac{Nd}{3\epsilon_0} \right) = NdE$$

$$P = \frac{NdE}{1 - \frac{Nd}{3\epsilon_0}}$$

W.K.T, $P = \epsilon_0 E (\epsilon_r - 1)$

$$\epsilon_0 E (\epsilon_r - 1) = \frac{NdE}{1 - \frac{Nd}{3\epsilon_0}}$$

$$1 - \frac{Nd}{3\epsilon_0} = \frac{Nd}{\epsilon_0 E (\epsilon_r - 1)}$$

$$1 = \frac{Nd}{\epsilon_0 (\epsilon_r - 1)} + \frac{Nd}{3\epsilon_0}$$

$$1 = \frac{Nd}{3\epsilon_0} \left(\frac{3}{\epsilon_r - 1} + 1 \right)$$

$$1 = \frac{Nd}{3\epsilon_0} \left(\frac{3 + \epsilon_r - 1}{\epsilon_r - 1} \right)$$

$$1 = \frac{Nd}{3\epsilon_0} \left(\frac{\epsilon_r + 2}{\epsilon_r - 1} \right)$$

$$\Rightarrow \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{Nd}{3\epsilon_0}$$

Q) A solid dielectric material has 4×10^{28} atoms/m³ if it shows electronic polarisability of 1.5×10^{-40} Fm² then calculate dielectric constant.

Sol $N = 4 \times 10^{28}$ atm/m³

$\alpha = 1.5 \times 10^{-40}$ Fm²

$\epsilon_r = ?$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1000}{4427}$$

$$4427\epsilon_r - 4427 = 1000\epsilon_r + 2000$$

$$3427\epsilon_r = 6427$$

$$\epsilon_r = \frac{6427}{3427}$$

$$\epsilon_r = 1.875$$

Q) At a temperature 27°C the dielectric constant of sulphur is 3.75. If the density of sulphur at 27°C is 2050 kg/m³ then calculate electronic polarisability of sulphur (At. wt of Sulphur is 32)

$$N = \frac{N_A \times \rho}{\text{At. wt}}$$

$$N = 3.85 \times 10^{28}$$
 atm/m³

$\epsilon_r = 3.75$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

$$\frac{2.75}{5.75} = \frac{(3.85 \times 10^{28})\alpha}{3 \times 8.854 \times 10^{-12}}$$

$$\alpha = \frac{11 \times 3.85 \times 10^{28}}{23 \times 3.85 \times 10^{28}}$$

$$\alpha = 3.29 \times 10^{-40} \text{ Fm}^2$$

Q) There are 1.6×10^{20} NaCl molecules / m^3 in a vapour. Find orientational polarisation at room temperature the vapour is subjected electric field of 5×10^5 V/m NaCl consist of Na & Cl separated by 0.25 nm

$$P = \frac{NM^2 E}{3K_B T}$$

$$N = 1.6 \times 10^{20}$$

$$E = 5 \times 10^5$$

$$K_B = 1.38 \times 10^{-23}$$

$$T = 300 \text{ K}$$

$$M = q \times d$$

$$= (1.6 \times 10^{-19}) \times (0.25 \times 10^{-9})$$

$$M = 4 \times 10^{-29}$$

$$M^2 = 1.6 \times 10^{-57}$$

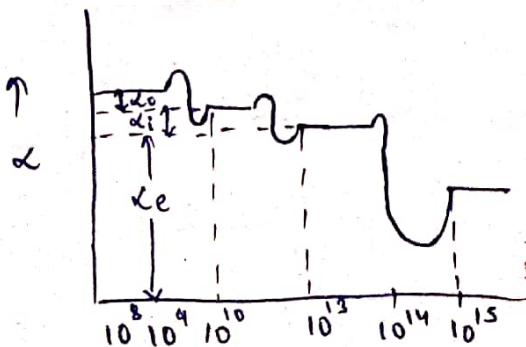
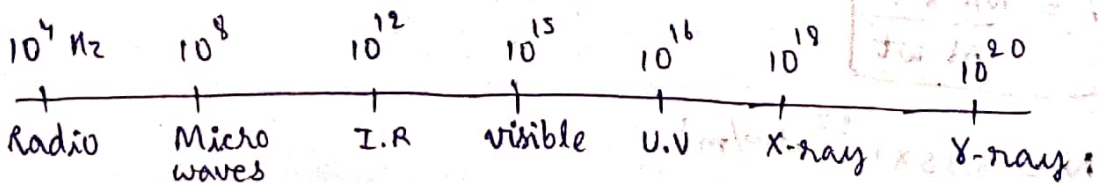
$$P = \frac{(1.6 \times 10^{20})(1.6 \times 10^{-57})(5 \times 10^5)}{3 \times 1.38 \times 10^{-23} \times 300}$$

$$P = 1.03 \times 10^{-11} \text{ cm}^2$$

$$a) C = \frac{\epsilon_0 A}{d}$$

Find capacitance

Frequency dependence Polarizability



$$P(t) = P \left(1 - \exp\left(-\frac{t}{\tau_r}\right) \right)$$

τ_r = Relaxation time

The time taken for polarisation process to reach maximum value is called relaxation time.

Electronic Polarizability

e^- - e^- - lighter particles - reacts fast - Polarisation time is less - Frequency is more - 10^{15} Hz - Visible region

Ionic Polarizability

i^- - ions are heavier than e^- - move slow - takes more time for polarisation - Frequency is less - 10^{13} Hz - I.R region

Orientational Polarizability

d_0 - orientation of dipole moments in the direction same as that of the applied field takes still more time - Polarisation time is more - Frequency is very less - 10^{10} Hz - Microwave region



... ..

... ..

$\left[\begin{matrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{matrix} \right] \left[\begin{matrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{matrix} \right] = \left[\begin{matrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{matrix} \right] \left[\begin{matrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{matrix} \right]$