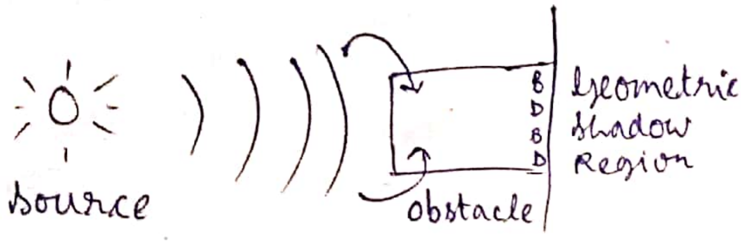
Diffraction

It is derived from the Latin word "diffractus" which means break into pieces.

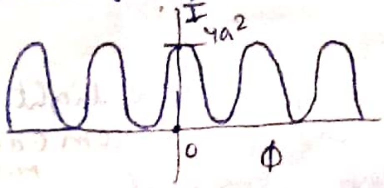
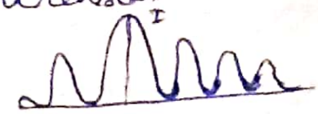


→ The phenomenon of bending of light rays around the corners of an obstacle and spreading of light waves into geometrical shadow region of an obstacle placed in a path of light is called diffraction.

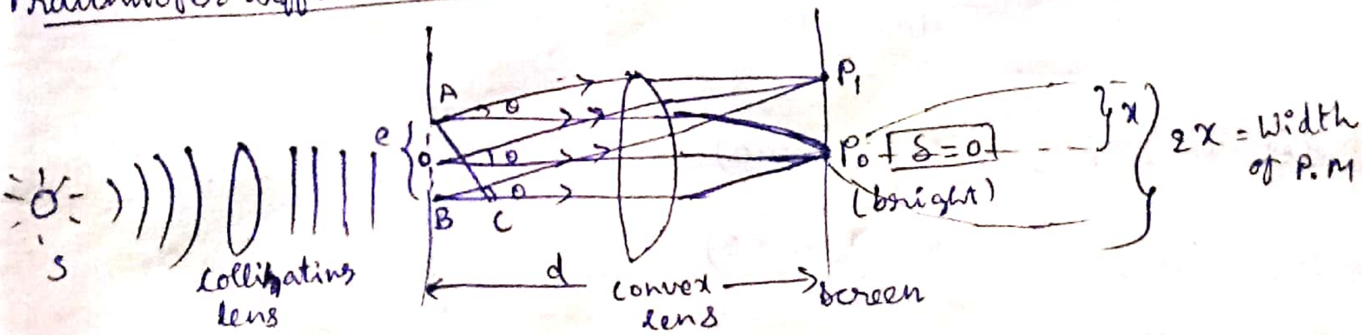
Condition :-
The size of an obstacle should be comparable to the wavelength of light source used.

Types of diffraction

Fresnel	Fraunhofer
1) Either a point source or illuminated narrow slit is used.	1) Extended source at infinite distance is used.
2) The source and screen should be at a finite distance from obstacle.	2) The source and screen should be at infinite distance from obstacle.
3) No lenses are used.	3) Convex lens are used.
4) Incident wavefront should be spherical or cylindrical.	4) Incident wavefront should be plane.
5) Study of diffraction is easy.	5) Study of diffraction is difficult.
6) ~~It~~ Diffraction can be studied only in the direction of propagation of light.	6) Diffraction can be studied in any direction of propagation of light.

Interference	Diffraction
1) It is due to superposition of two different wavefronts originating from two coherent (or) different sources.	1) It is due to superposition of secondary wavelets originating from different parts of same wavefront.
2) All the fringes have same width.	2) Fringe width decreases as order increases.
	
3) All the bright fringes have same intensity.	3) All the bright fringes will not have same intensity
4) All the dark fringes have zero intensity.	4) All the dark fringes will not have zero intensity.

* Fraunhofer diffraction due to single slit



$BC = \text{Path diff.} (\delta)$

* let a monochromatic light ray is passed through the collimating lens in order to get the plane wave fronts, which passed through the slit AB of width 'e'. The diffracted light rays through the slits are focussed on the screen by means of convex lens.

* Huygen's concept – Each and every point in the primary wave-front will act as the secondary source from which we get secondary wavelets. These secondary wavelets will travel in all the directions.

* The secondary wavelets which are travelling normal to the slit are focussed at the point P_0, hence the point P_0 is bright ($\delta = 0$)

* The secondary wavelets which are travelling at an angle θ are focussed at a point P_1

* To know whether the point P_1 is bright or dark we have to calculate the path difference. To calculate the path difference

we have to draw the normal.

$BC = ?$

From ΔABC,

$$\sin\theta = \frac{BC}{AB}$$

Path diff = $\boxed{BC = e\sin\theta}$

$$\phi = \frac{2\pi}{\lambda}(\delta)$$

$$\boxed{\phi = \frac{2\pi}{\lambda}(e\sin\theta)}$$

⇒ let the slit AB is divided into 'n' equal parts, (a = amplitude of light rays coming from each part)

⇒ The phase diff. b/w the light waves that are coming from each part is

$$\phi = \frac{2\pi}{\lambda} e\sin\theta$$

$$1 \text{ part} = \frac{1}{n}\left[\frac{2\pi}{\lambda} e\sin\theta\right] = d$$

→ From the vector addition of amplitude, the resultant amplitude is given as $\boxed{R = \frac{a \sin nd/2}{\sin d/2}}$

~~Let~~ ⇒ $$R = \frac{a\sin\left(\frac{\pi}{\lambda} e\sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda} e\sin\theta\right)}$$

Let $\frac{\pi}{\lambda} e\sin\theta = \alpha$

$$R = \frac{a\sin\alpha}{\sin\frac{\alpha}{n}}$$

As $\frac{\alpha}{n}$ is very small $\left(\sin\frac{\alpha}{n} \approx \frac{\alpha}{n}\right)$

$$R = \frac{a\sin\alpha}{\frac{\alpha}{n}}$$

$$R = \frac{na\sin\alpha}{\alpha}$$

$$\boxed{R = \frac{A\sin\alpha}{\alpha}} \qquad \boxed{I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2} \qquad \left(\because I = (amp)^2\right)$$

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Principal Maxima :-

$$R = A\left(\frac{\sin\alpha}{\alpha}\right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$R = \frac{A}{\alpha}\left(\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \dots\right)$$

$$R = \frac{A}{\alpha}\alpha\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \dots\right]$$

$\boxed{\alpha = 0}$ $\Rightarrow \frac{\pi}{\lambda} e\sin\theta = 0$

$R = A$ $\boxed{e\sin\theta = 0}$

$\boxed{I = R^2 = A^2}$ $\sin\theta = 0$

$\theta = 0$

$\Rightarrow$ Intensity will be maximum when amplitude will be maxium. To get amaximum amplitude, negative terms are vanished by putting $\alpha = 0$

Minima :-

$I = 0$

$$A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$\sin\alpha = 0$

$\alpha = 0, \pm\pi, \pm 2\pi, \dots$

$\boxed{\alpha = \pm m\pi}$

$\frac{\pi}{\lambda} e\sin\theta = \pm m\pi$

$\boxed{e\sin\theta = \pm m\lambda}$

where $m = 1, 2, 3, \dots$

If $m = 0 \Rightarrow e\sin\theta = 0 \Rightarrow \theta = 0$ which corresponds to principal maxima

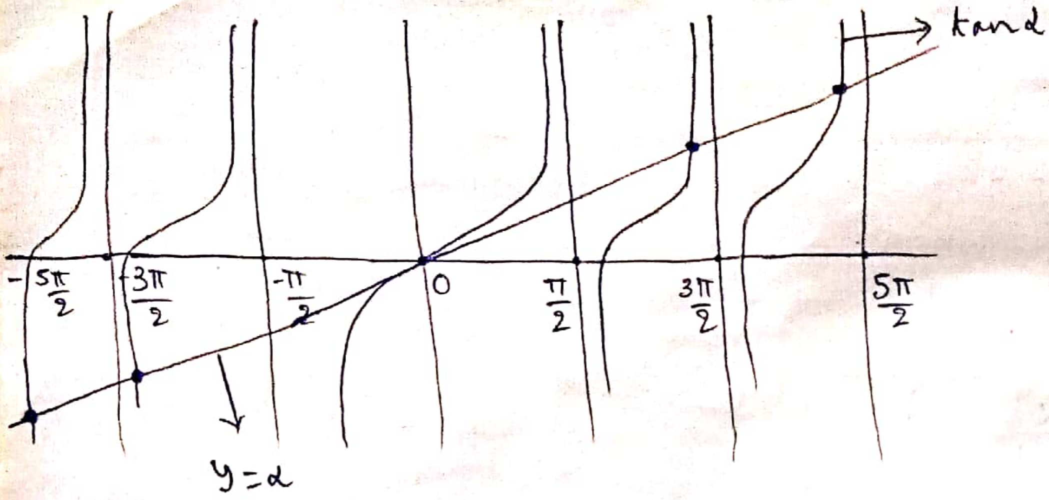
$\Rightarrow$ We get minimal positions on either side of the maxima

Secondary maxima :-

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad -(1)$$

$$\frac{dI}{d\alpha} = A^2 \cdot 2\left(\frac{\sin\alpha}{\alpha}\right) \times \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

Now, put $\alpha\cos\alpha - \sin\alpha = 0$ $\Rightarrow \boxed{\alpha = \tan\alpha}$



Points of intersection

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots$$ (Here, we should not consider $\alpha = 0$, because it belongs to principal maxima)

$$\frac{\pi}{\lambda} e \sin\theta = \pm(2n+1)\frac{\pi}{2}$$

$$\boxed{e\sin\theta = \pm(2n+1)\frac{\lambda}{2}}$$

Substituting "α" values in (1)

(i) $\alpha = 0$

$\boxed{I = A^2}$ (Principal Maxima)

(ii) $\alpha = \frac{3\pi}{2}$

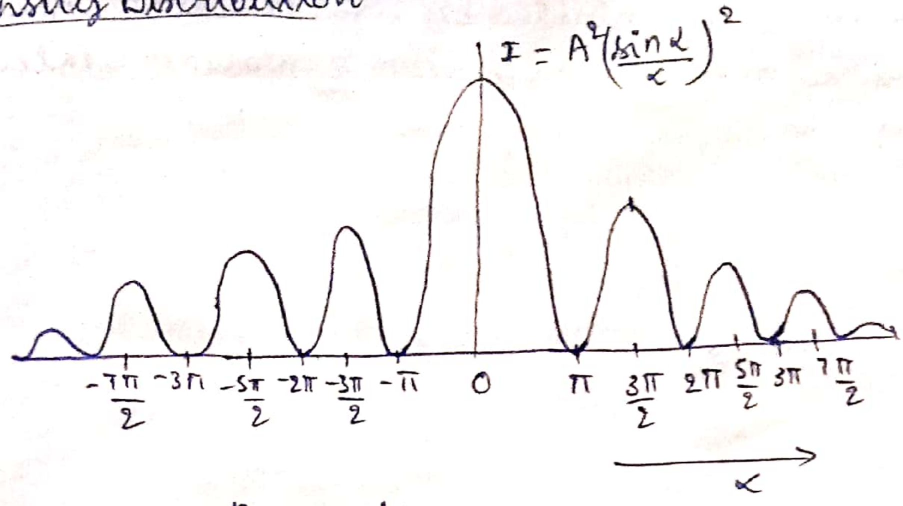
$$I = A^2\left(\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}\right)^2 \simeq \frac{A^2}{22} = 0.045A^2 \text{ (1}^{st}\text{ secondary max.)}$$

(iii) $\alpha = \frac{5\pi}{2}$

$$I = A^2\left(\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}}\right)^2 \simeq \frac{A^2}{62} = 0.016A^2 \text{ (2}^{nd}\text{ secondary max.)}$$

→ Most of the intensity is concentrated at principal maxima and rest is distributed among ~~other~~ secondary maximas.

Intensity Distribution



$\alpha = 0 \rightarrow$ Principal max

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots \ldots \rightarrow$ Minima

$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots \rightarrow$ Secondary Max

Expression for width of Principal Maxima

We know that,

$$e\sin\theta = \pm m\lambda$$

$m=1$ $\boxed{\sin\theta = \frac{\lambda}{e}}$ — (2)

In ΔOP_oP_1,

$$\sin\theta = \frac{opp.}{hyp} = \frac{P_oP_1}{OP_1} = \frac{x}{d}$$

(d = distance from slit to the screen)
x = Half width of the P.M

$\boxed{\sin\theta = \frac{x}{d}}$ — (3)

$$\frac{\lambda}{e} = \frac{x}{d} \quad (\because \text{From (2) \& (3)})$$

$$\boxed{x = \frac{\lambda d}{e}}$$

$$\boxed{2x = \frac{2\lambda d}{e}}$$

As the convex lens near to the ~~screen~~ slit,

$\boxed{x = \frac{\lambda f}{e}}$ f = focal length

Conclusion :- From the above eq. it is clear that when the slit width decreases, then the fringe width increases i.e, when slit width is narrow then fringe width becomes wider. When the source of higher wavelength is used then the fringes are wider.

Q) The slit of width 1.5 mm is illuminated by light of wavelength 500 nm & diffraction pattern is observed on screen 2 m away. Calculate the width of central max.

Sol

$e = 1.5\,mm$

$e = 1.5 \times 10^{-3}\,m$

$\lambda = 500\,nm$

$\lambda = 500 \times 10^{-9}\,m$

$d = 2\,m$

$2x = ?$

$$2x = \frac{2\lambda d}{e} = \frac{2 \times 500 \times 10^{-9} \times 2}{1.5 \times 10^{-3}} = \frac{2 \times 10^{-6}}{15 \times 10^{-4}} = \frac{2}{15} \times 10^{-2}$$

$= 1.33 \times 10^{-3}\,m$

$= 1.33\,mm$

Q) A screen is placed 2 m away from narrow slit. Find the slit width if 1st minima lies 5 mm on either side of central maxima, when a plane wave of 500 nm is incident on slit

$d = 2\,m$

$x = 5 \times 10^{-3}\,m$

$\lambda = 500 \times 10^{-9}\,m$

$$x = \frac{\lambda d}{e} \Rightarrow e = \frac{\lambda d}{x} = \frac{500 \times 10^{-9} \times 2}{5 \times 10^{-3}} = 2 \times 10^{-4}$$

$= 0.2 \times 10^{-3}\,m$

$= 0.2\,mm$

Q) A lens of focal length 0.4 m and slit width of 0.2 mm are used to obtain diffraction pattern. Calculate the distance of 1st dark band and width of central maxima if wavelength of light is 500 nm

$f = 0.4\,m = \cancel{0.4 \times 10^{3}\,mm}$

$e = 0.2 \times 10^{-3}\,m$

$\lambda = 500 \times 10^{-9}\,m$

$$\boxed{x = \frac{\lambda f}{e}} \Rightarrow x = \frac{500 \times 10^{-9} \times 0.4}{0.2 \times 10^{-3}} = \frac{5 \times 10^{-8} \times 4}{2 \times 10^{-4}} = 10 \times 10^{-4}$$

→ For 1st dark band

$= 1 \times 10^{-3}\,m$

$\boxed{x = 1\,mm}$

$\boxed{2x = 2\,mm}$

→ For P.M

1) Calculate the angles at which 1st dark band and next bright band are formed in the Fraunhofer diffraction pattern of a slit 0.3mm width ($\lambda = 5890 A^\circ$)

$e = 0.3\ mm \quad = 0.3 \times 10^{-3}\ m$

$\lambda = 5890 \times 10^{-10}\ m$

For 1st dark band

$e \sin\theta = \pm\lambda$

$\boxed{\theta = \sin^{-1}\left(\frac{\lambda}{e}\right)}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{5890 \times 10^{-10}}{0.3 \times 10^{-3}}\right)$

$\boxed{\theta = 0.1124}$

For 1st bright band

$e \sin\theta = \pm \frac{3\lambda}{2}$

$\boxed{\theta = \sin^{-1}\left(\frac{3\lambda}{2e}\right)}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{3 \times 5890 \times 10^{-10}}{2 \times 0.3 \times 10^{-3}}\right)$

$\boxed{\theta = 0.1687}$

★

2) ~~5d~~ Find the half angular width of central max. in Fraunhofer diffraction of a slit of width 12×10^{-5} cm. When the slit is illuminated by monochromatic light of wavelength 6000 A°

$e = 12 \times 10^{-5}$ cm

$\lambda = 6000 \times 10^{-8}$ cm

$\boxed{\sin\theta = \frac{\lambda}{e}} \rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{e}\right) = \sin^{-1}\left(\frac{6 \times 10^{-5}}{\frac{12 \times 10^{-5}}{2}}\right) = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = 30^\circ}$

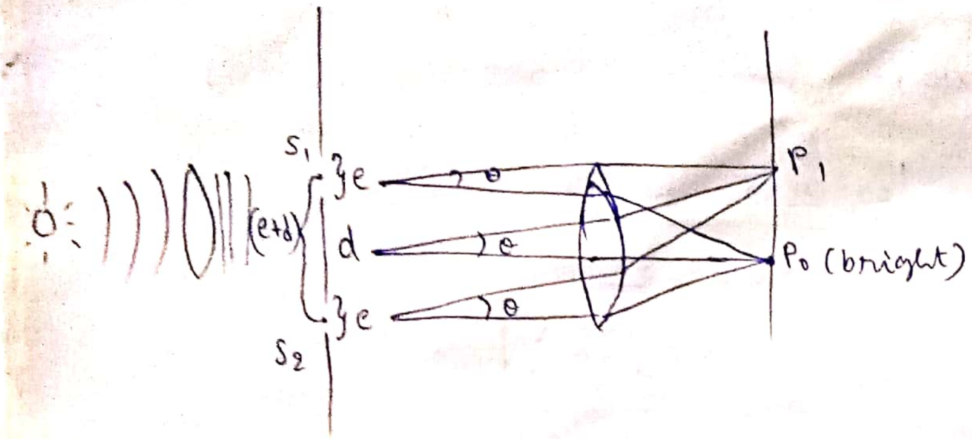
a) Find the half angular width of central maxima in Fraunhofer diffraction using a slit of 1 μm. When the slit is illuminated by the light of wavelength 600nm.

$e = 1 \times 10^{-6}$ m

$\lambda = 600 \times 10^{-9}$

$\theta = \sin^{-1}\left(\frac{\lambda}{e}\right) = \sin^{-1}\left(\frac{6 \times 10^{-7}}{1 \times 10^{-6}}\right) = \sin^{-1}(6 \times 10^{-1}) \Rightarrow \boxed{\theta = 36^\circ 52'}$

Diffraction due to double slit (Fraunhofer)



⇒ Let the light rays are passed through the double slit whose width are 'e' which are separated by a distance of 'd'. The distance from the middle points of the slits is $(e+d)$.

⇒ We know that, from diffraction due to single slit $\boxed{R = \frac{A\sin\alpha}{\alpha}}$

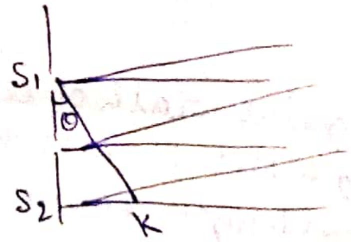
∴ The secondary wavelets that are coming from each slit has an amplitude of $\frac{A\sin\alpha}{\alpha}$. Now, these secondary wavelets whose amplitude is $\frac{A\sin\alpha}{\alpha}$ interfere and reaches the point P_1. To know the point P_1 is bright or dark we have to calculate the path difference.

To calculate the path difference, let us draw a normal (S_1K)

Consider ΔS_1S_2K

Path diff. $S_2K = ?$

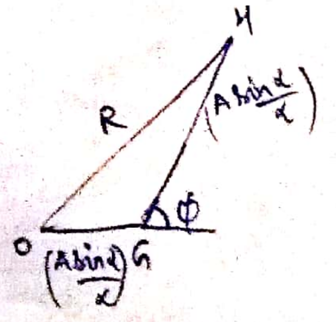
$$\sin\theta = \frac{S_2K}{S_1S_2}$$



$$\boxed{S_2K = (e+d)\sin\theta}$$

Phase diff. $\boxed{\phi = \frac{2\pi}{\lambda}(e+d\sin\theta)}$

To calculate the resultant amplitude, we are using vector addition method.



$$R^2 = OG^2 + GH^2 + 2(OG)(GH)\cos\phi$$

$$R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 + A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 + 2A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos\phi$$

$$= A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[1+1+2\cos\phi\right]$$

$$= 2A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[1+\cos\phi\right]$$

$$= 2A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(2\cos^2\phi/2\right)$$

$$R^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\frac{\phi}{2}$$

$$= 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\left(\frac{\pi}{\lambda}(e+d\sin\theta)\right)$$

let $\boxed{\beta = \frac{\pi}{\lambda}(e+d\sin\theta)}$

$$\Rightarrow \boxed{R^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta} = I$$

- The resultant intensity equation is the combination of two factors. 1) $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$, which represents diffraction due to single slit

2) $\cos^2\beta$, which represents interference due to double slit

Diffraction effect :-

$\Rightarrow$ Principal Maxima $\longrightarrow \alpha = 0$, $\boxed{e\sin\theta = 0}$

$\Rightarrow$ Minima $\longrightarrow \alpha = \pm\pi, \pm 2\pi \ldots m\pi$

$\boxed{e\sin\theta = \pm m\lambda}$

$\Rightarrow$ Secondary Maxima $\longrightarrow \alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots$

$\boxed{e\sin\theta = \pm(2n+1)\frac{\lambda}{2}}$

Interference Effect :-

$\Rightarrow$ Maxima will occur when $\cos^2\beta = 1$

$\Rightarrow$ Minimum intensity will occur when $\cos^2\beta = 0$

For maxima

$\beta = 0, \pm\pi, \pm 2\pi, \ldots$

$\boxed{\beta = \pm n\pi}$

$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$

$\boxed{(e+d)\sin\theta = \pm n\lambda}$

For minima

$\beta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\boxed{\beta = (2n+1)\frac{\pi}{2}}$

$\frac{\pi}{\lambda}(e+d)\sin\theta = (2n+1)\frac{\pi}{2}$

$\boxed{(e+d)\sin\theta = (2n+1)\lambda/2}$

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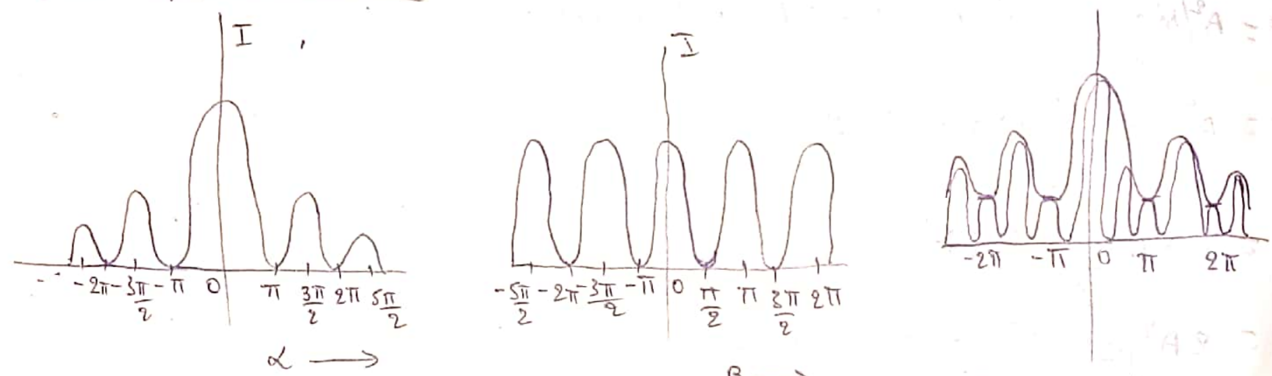
Intensity Distribution

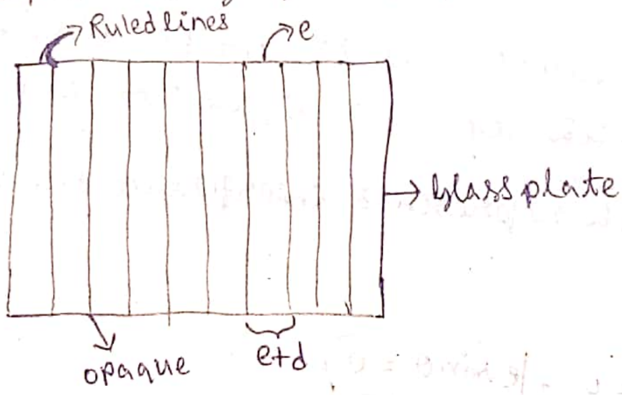


Diffraction   Interference   Resultant

⇒ From the figure, it is clear that minima is not equal to zero, still we have some minimum intensity due to interference effect.

Diffraction grating :-

An arrangement consisting of large no. of parallel slits of same width and separated by equal opaque space is called diffraction grating.



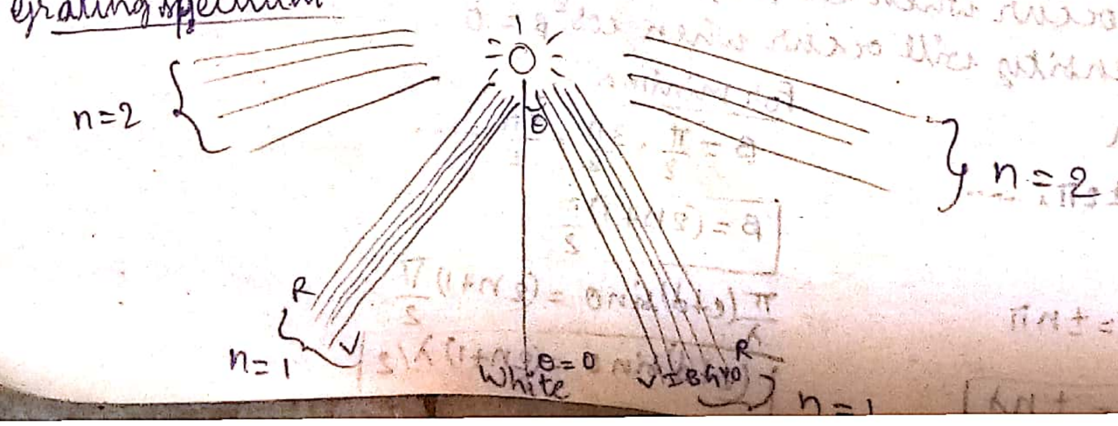
Grating element :-

The combined width of the slit and ruled line is called grating element (e+d)

$N(e+d) = 1"$; N = no. of lines

$$\boxed{N = \frac{1"}{e+d}} \Rightarrow \boxed{N = \frac{2.54\,(\text{cm})}{e+d}}$$

Grating spectrum



⇒ The condition for principal maxima in grating is $\boxed{(e+d)\sin\theta = n\lambda}$

(i) For the particular order, angle of diffraction is different for different wavelengths.

(ii) For the particular wavelength, angle of diffraction is different for different orders.

(iii) Longer the wavelength, greater the angle of diffraction.

(iv) At centre $\theta = 0$ maxima of all the colours coinside with each other and the resultant colour is same as that of the source used.

(v) Most of the intensity is concentrated at the centre and rest is distributed among other orders.

(vi) Spectral lines are dispersed more and more as we go for higher orders.

(vii) Spectra of different orders are situated symmetrically on either side of the zero order.

(viii) As there are large no. of lines in the grating, maxima appears to be sharp, bright parallel lines which is termed as spectral lines.

(ix) Maximum no. of orders possible in the grating is

$$(e+d)\sin\theta = n\lambda$$

$$\theta = 90^\circ$$

$$(e+d) = n\lambda$$

$$1 = \frac{n\lambda}{e+d}$$

$$1 = Nn\lambda$$

$$\boxed{n_{max} = \frac{1}{N\lambda}}$$

Q) A plane transmission grating having 4250 lines (per cm) illuminated with a sodium light normally. In 2nd order spectrum the 2nd order lines are deviated by 30°. What is the wavelength of spectral line?

Sol

$N = 4250$ lines/cm

$\theta = 30^\circ$

$n = 2$

$\lambda = ?$

$$(e+d)\sin\theta = n\lambda$$

$$\sin\theta = Nn\lambda$$

$$\frac{1}{2} = 4250 \times 2 \times \lambda$$

$$\Rightarrow \frac{1}{4} \times \frac{1}{4250} = \lambda = 5.882 \times 10^{-5}\ \text{cm}$$

$$= \cancel{5882 \times 10^{-8}}$$

$$\lambda = 5.882 \times 10^{-5} \times 10^{8}$$

$$\boxed{\lambda = 5.882 \times 10^{+3} A^\circ}$$

$$\boxed{\lambda = 5882 A^\circ}$$

Q) A plane grating having 10520 lines/cm is illuminated with light having a wavelength of 5×10^{-5} cm at normal incidence. How many orders will be visible in a grating spectra

$$n = \frac{1}{N\lambda}$$

$$n = \frac{1}{10520 \times 5 \times 10^{-5}} = \frac{10^5}{5 \times 10520} = 1.9 \approx 2$$

$$\boxed{\therefore n \approx 2}$$

Q) A grating has 6000 lines/cm. Find the angular separation b/w two wavelengths of 500 nm and 510 nm in the third order.

Sol: $\theta_2 - \theta_1 = ?$

$\lambda_1 = 500 \times 10^{-9}$ m

$\lambda_2 = 510 \times 10^{-9}$ m

$N = 6000$ lines/cm

$= 6000 \times 10^{+2}$

$N = 6 \times 10^5$

$$\boxed{\sin\theta_1 = Nn\lambda_1}$$

$\sin\theta_1 = 6 \times 10^5 \times 3 \times 5 \times 10^{-7}$

$= 90 \times 10^{-2}$

$\sin\theta_1 = 0.9$

$\theta_1 = \sin^{-1}(0.9)$

$$\boxed{\theta_1 = 64.15}$$

$$\boxed{\sin\theta_2 = Nn\lambda_2}$$

$\sin\theta_2 = 6 \times 10^5 \times 3 \times 510 \times 10^{-9}$

$= 9180 \times 10^{-4}$

$= 0.9180$

$\theta_2 = \sin^{-1}(0.9180)$

$$\boxed{\theta_2 = 66.63}$$

$$\boxed{\theta_2 - \theta_1 = 2.48}$$

Q) Light from a sodium vapour lamp normally falls on a grating of 2 cm having 10000 lines. Find the angular separation of two line of sodium whose wavelengths are 5890 A° & 5896 A° in the 1st order spectrum

Sol:- $N = 10,000$ lines/2cm $\qquad \lambda_1 = 5890 \times 10^{-10}$ m

$N = 5000$ lines/cm $\qquad \lambda_2 = 5896 \times 10^{-10}$ m

$N = 5000 \times 10^2$ lines/m

$\sin\theta_1 = Nn\lambda_1$

$= 5 \times 10^5 \times 1 \times 5890 \times 10^{-10}$

$= 5 \times 5890 \times 10^{-5}$

Q) A monochromatic light of wavelength 6.56×10^{-7} m incident normally on a grating of 2cm wide, the 1st order spectrum is produced at an angle of $18° 14'$ from the normal. Calculate the total no. of lines in grating.

$$\boxed{N = \frac{\text{Total no. of lines (M)}}{\text{width}}}$$

$N = \frac{1}{n\lambda} = \frac{1}{6.56 \times 10^{-7}} = \frac{10^7}{6.56}$

$N = 1524390.24$

$M = N \times \text{width}$

$\cancel{= 1524390.24 \times 2}$

$\cancel{= 3048780.}$

$\sin\theta = Nn\lambda$

$N = \frac{\sin\theta}{n\lambda}$

$N = 476962.7$

$= 476962.7 \times 2$

$\boxed{M = 9540}$

Q) Mercury light is normally incident on grating, diffraction angle in 1st order spectrum for green is 5460 A° at 20° find no. of lines/cm of a grating.

$$\boxed{\sin\theta = Nn\lambda}$$

$\sin 20° = N(1) 5460 \times 10^{-8}$

$N = \frac{\sin 20°}{5460 \times 10^{-8}} = \frac{0.34}{5460 \times 10^{-8}} = 6227$

Q) Calculate the possible order of spectrum with a plane transmission having 18000 lines/inch. When a light of wavelength 4500 A° is used.

$\lambda = 4500 \times 10^{-8}$ cm

$N = \frac{18000}{2.54}$ lines/cm $= 7086$