UNIT-III

ELECTROMAGNETIC WAVES

Electromagnetic waves or EM waves are waves that are created as a result of vibrations between an electric field and a magnetic field. In other words, EM waves are composed of oscillating magnetic and electric fields. The science of electromagnetism was developed by many scientists but mainly we mention two of them were more significant. One of the scientist was Michael Faraday (1791-1897). Another one was James Clerk Maxwell (1831-1879) .Maxwell's equations play the same role in electromagnetism as that Newton's laws in mechanics.

3.1 Scalar and Vector fields:

A continuous function of the position of a point in a region of space is called a point function. The region of space in which it specifies a physical quantity is known as a field. These fields are classified into two groups

1. Scalar Field
2. Vector Field

1. Scalar Field : If a scalar physical quantity is assigned to each point in a space then we have a scalar field in that region of space (or) It is defined as that region of space, whose each point is associated with a scalar point function.

Example: Temperature of a heated body, potential around a charge, distribution of mass of a body.

2. **Vector field :** A vector field is specified by a continuous vector point function having magnitude and direction, both of which change from point to point, in the given region of field. The method of presentation of a vector field is called vector lines or lines of surfaces. The tangent at a vector line gives the direction of the vector at the point.

Example: Electric field, Magnetic field, Gravitation field etc.

3.2 Gradient (or) Gradient of a Scalar field

The gradient is a differential operator by means of which we can associate a vector field with a scalar field.

For example the intensity of electric field E (Vector) is the gradient of potential V (Scalar) with a negative sign.

$$E = -\text{ grad } V$$

The negative sign indicates that the direction of field intensity is opposite to the direction of increase of potential.

Let S(x, y, z) be a scalar point function depending on the three cartesian coordinates in space, then

grad S = $i\frac{\partial S}{\partial x}$+ j$\frac{\partial S}{\partial y}$ + k $\frac{\partial S}{\partial z}$

grad S = ∇.S

Where ∇=$i\frac{\partial}{\partial x}$+ j$\frac{\partial}{\partial y}$ + k $\frac{\partial}{\partial z}$

Problem:

If S=S(x, y, z)=$x^2 - x^2y + xy^2z^2$ then find the value of grad S at the point (2, -1, 3)

Solution: grad S= ∇.S

=($i\frac{\partial}{\partial x}$+ j$\frac{\partial}{\partial y}$ + k $\frac{\partial}{\partial z}$) ($x^2 - x^2y + xy^2z^2$)

= $i\frac{\partial}{\partial x}$($x^2 - x^2y + xy^2z^2$)+ j$\frac{\partial}{\partial y}$ ($x^2 - x^2y + xy^2z^2$) + k $\frac{\partial}{\partial z}$ ($x^2 - x^2y + xy^2z^2$)

= I ($2x - 2xy + y^2z^2$)+j ($-x^2 + 2xyz^2$)+k ($2zxy^2$)

Substituting the value of x, y, z in the above equation we will get

grad S=17i -40j-12k

3.3 Divergence (or) Divergence of a vector Field:

The scalar or dot product of operator ∇ with a vector is called as divergence.

If A is a vector (A= $iA_x + jA_y + kA_z$) then div A = ∇.A

The divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from that point. Divergence is a Scalar.

Let A be a vector function differentiable at each point (x, y, z) in a region of space, then the divergence of A is given by

div A = ∇.A =($i\frac{\partial}{\partial x}$+ j$\frac{\partial}{\partial y}$ + k $\frac{\partial}{\partial z}$). ($iA_x + jA_y + kA_z$)

= $\frac{\partial A_x}{\partial x}$+ $\frac{\partial A_y}{\partial y}$ + $\frac{\partial A_z}{\partial z}$

Physical Significance of Divergence :

- If A represents the velocity of a fluid then div A gives the rate of flow of fluid at that point per unit volume.
- div A is positive at that point it means the fluid is undergoing expansion.
- div A is negative at that point it means the fluid is undergoing contraction.
- div A =0 means the fluid entering and leaving the element is same. The vector satisfying this condition is called solenoidal.

Problem :

If A=$\boldsymbol{iy + j(x^2 + y^2)}$+$\boldsymbol{k(yz + zx)}$then find div A at point (1, -2 ,3).

div A= ∇.A

$= (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}).(iy + j(x^2 + y^2) + k(yz + zx))$

$= \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(x^2 + y^2) + \frac{\partial}{\partial z}(yz + zx)$

$=0+2y+ y + x = 3y + x$ =3(-2)+1=-6+1=-5.

3.4 Curl of a Vector field :

The property of curl is shown by fields for which line integral round of any closed path is non-zero and cannot be derived as the gradient of scalar field.

" The curl of a vector field is defined as the maximum line integral of the vector per unit area." It is a vector quantity and direction is normal to the area.

If A is a vector function differentiable at each point (x, y, z) in a region of space then the curl (rotation) of A is expressed by the cross product of ∇ and A. It is a vector quantity and the direction is normal to the area.

Curl A= ∇ x A

$$\text{curl } A = \nabla \times A \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ax & Ay & Az \end{vmatrix}$$

Physical Significance of Curl :

- The general meaning of curl is rotation
- Curl A is zero that means there is no rotation or moving with same velocity.
- Curl A is non zero it means rotation is attached with that vector.

3.5 Stoke's Theorem for Curl

Statement:

Stoke's theorem states that the line integral of a vector field 'A' around a closed curve is equal to the surface integral of the curl of A taken over the surface 'S' surrounded by the closed curve.

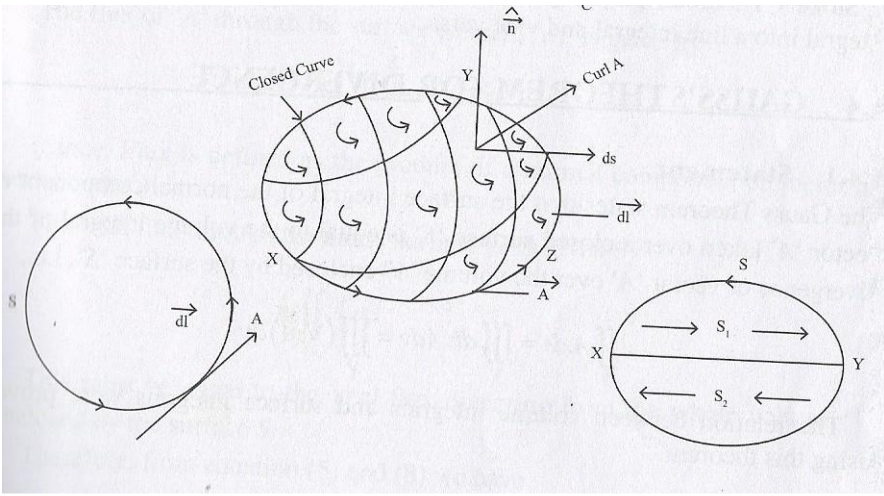
It is expressed as

$$\oint_C A.dl = \iint_S \text{curl} A.ds = \iint_S (\nabla \times A)\, ds.$$

Theorem:

Let us consider surface 'S' is enclosed in a vector field 'A' as shown in the figure, in which a closed curve XYZ represents the boundary of surface S.

The line integral of 'A' around the curve XYZ is $\oint_C A.dl$



Let the surface 'S' is divided into large number of square loops. 'ds' be the area enclosed by each small loop, and n be unit positive outward normal upon small loop 'ds'. The vector area of the element is

$$n.ds=dS \text{ ------------(1)}$$

The curl of a vector field at any point is the maximum line integral of the vector compared per unit area , along the boundary of an infinitesimal area at the point. Then, the line integral of A around the boundary of 'ds' is

curl A dS-------------(2)

It is applied to all surface elements and hence, sum of line integrals of 'A' around the boundaries of all the area elements can be written as

$$\iint_S \; curl\; A\; ds \text{--------(3)}$$

Which is surface integral of 'A'.

From the figure, we can understand that the line integrals along the common sides of the continuous elements mutually cancel because they are traversed in the opposite direction.

The sides of the elements which lie in the closed curve of the surface contribute to the line integral. The equation (3) gives the sum of the line integrals on the boundary line of the curve.

$$\oint_C A.dl = \iint_S \mathrm{curl} A.ds = \iint_S (\nabla \times A)\, ds.$$

3.6 Gauss's Theorem for Divergence

Statement:

The Gauss theorem states that the surface integral of the normal component of vector 'A' taken over a closed surface 'S' is equal to the volume integral of the divergence of vector 'A' over the volume 'V' enclosed by the surface 'S'.

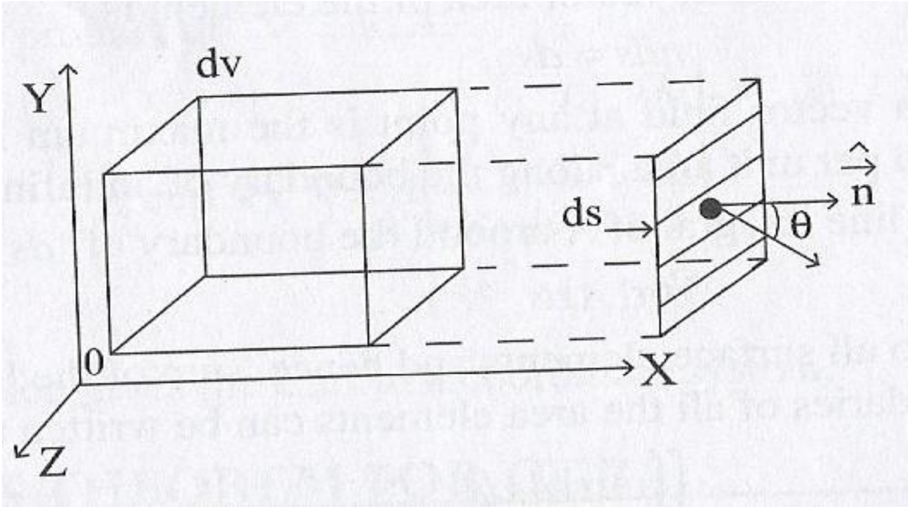
$$\iint_S A.ds = \iiint_V divAdv = \iiint_V (\nabla.A)\, dv$$

Theorem:

Let us consider a surface 'S' in a vector field 'A' as shown in the figure. Let 'V' be the volume enclosed in the surface and the volume is divided into a large number of cubical volume elements 'dv'.

'div A' represents the amount of flux diverging per unit volume and hence, the flux diverging

From the element of volume 'dv' will be 'div A dv'.



Hence, the total flux coming from the entire volume can be written as

$$\iiint_v divA dv$$

For a field vector 'A' and outward normal n at an angle 'θ' , the 'A' component along n can be written as

$$A \cos \theta = A.n$$

The flux of A through the surface element 'ds' is given by

$$(A.n)\, ds = A.ds$$

The total flux through the entire surface 'S' is given by

$$\iint_s A . ds$$

$$\iint_s A . ds = \iiint_v div\, A\, dv$$

<u>3.7 Maxwell's Equations</u> :

Maxwell's equations are the basic equations of electromagnetism which are a collection of Gauss's law for electricity, Gauss's law for magnetism, Faraday's law of electromagnetic induction and Ampere's law for currents in conductors. Maxwell equations give a mathematical model for electric, optical, and radio technologies, like power generation, electric motors, wireless communication, radar, and, Lenses, etc. These Equations explain how magnetic and electric fields are produced from charges.

These equations are part of the comprehensive and symmetrical theory of electromagnetism, which is essential to understand electromagnetic waves, optics, radio and TV transmission, microwave ovens and magnetically levitated trains.

Maxwell's First Equation:

Maxwell's first law of electromagnetism is also known as Gauss's law for electricity.

Gauss's law states that the net flux of an electric field in a closed surface is directly proportional to the enclosed electric charge

$$\oint E.ds = \frac{q}{\varepsilon_0}$$

$$\iint E.ds = \frac{1}{\varepsilon_0}\iiint \rho.dv$$

Applying Gauss divergence theorem

$$\iiint (\nabla.E)\,\mathrm{dv} = \frac{1}{\varepsilon_0}\iiint \rho.dv \Rightarrow \nabla.E = \frac{\rho}{\varepsilon_0}$$

Maxwell's Second Equation:

Maxwell's second law of electromagnetism is also known as Gauss's law for magnetism

The Gauss's law for magnetism states that net flux of the magnetic field through a closed surface is zero because monopoles of a magnet do not exist.

$$\oint B.ds = 0$$

Applying Gauss divergence theorem $\iiint (\nabla.B)\mathrm{dv} = 0$

$$\nabla.B = 0$$

Maxwell's Third Equation:

Maxwell's Third law of electromagnetism is also known as Faraday's law of electromagnetic induction

Faraday's law of electromagnetic induction

The instantaneous emf induced in the circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

$$\varepsilon = -\frac{\partial \emptyset_B}{\partial t}$$

From this $\oint E.dl = -\frac{\partial}{\partial t}\oint B.dA$

It is the integral form of Maxwell's Third equation

Applying Stoke's Theorem to the integral form of third equation $\oint E.dl = -\frac{\partial}{\partial t}\oint B.dA$

$$\oint(\nabla \times E)\ dA = -\frac{\partial}{\partial t}\oint B.dA$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is Maxwell third law of electromagnetism.

Maxwell's Fourth Equation:

Maxwell's fourth law is also known as Ampere's law.

Ampere's Law: This law states that the amount of work done in carrying a unit magnetic pole along a closed path linked with the current is μ_0 times of the current.$\oint B.dl = \mu_0 i$

We know that $j = \frac{i}{A} = \frac{i}{ds} \Rightarrow i = j.\ ds$

$$\oint B.dl = \mu_0 \iint j.\ ds$$

Applying Stoke's theorem to the above equation

$$\iint(\nabla \times B)\ \mathrm{ds} = \mu_0 \iint j.\ ds$$

From this we can write $\nabla \times B = \mu_0 j$

$$\frac{1}{\mu_0}(\nabla \times B) = j \qquad \left(\frac{B}{\mu_0} = H\right)$$

$$\nabla \times H = j$$

Taking the divergence of both sides of equation $\nabla.(\nabla \times H) = \nabla.j$

Since the divergence of the curl of a vector is zero

$$\nabla.j = 0$$

It shows that net flux current out of any closed surface is zero, which is equation of continuity for steady fields and does not apply to time varying fields.

From Gauss's law $\nabla . E = \frac{\rho}{\varepsilon_0}$

Differentiating above equation

$\nabla . \frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial t}$ (or) $\varepsilon_0\ \nabla . \frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$

Adding $\nabla . j$ on both sides

$\nabla . j + \varepsilon_0\ \nabla . \frac{\partial E}{\partial t} = \nabla . j + \frac{\partial \rho}{\partial t}$

According to equation of continuity $\nabla . j + \frac{\partial \rho}{\partial t} = 0$

$\nabla . j + \varepsilon_0\ \nabla . \frac{\partial E}{\partial t} = 0$

$\nabla (j + \varepsilon_0 \frac{\partial E}{\partial t}) = 0$

From the displacement vector in free space $D = \varepsilon_0 E$

$\nabla (j + \varepsilon_0 \frac{\partial E}{\partial t}) = 0$

$\nabla (j + \frac{\partial D}{\partial t}) = 0$

Replacing j by $j + \frac{\partial D}{\partial t}$ in equation (12)

$$\nabla \times H = j + \frac{\partial D}{\partial t}$$

(or)

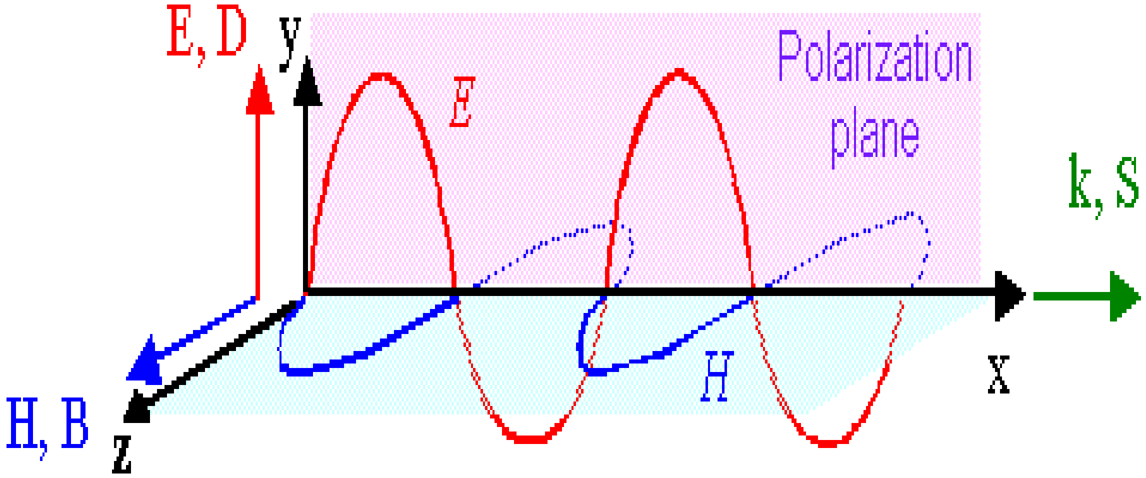
$$\nabla \times B = \mu_0 (j + \varepsilon_0 \frac{\partial E}{\partial t})$$

3.8 Poynting Theorem :

- In electrodynamics, Poynting's theorem is a statement of conservation of energy for the electromagnetic field, in the form of a partial differential equation developed by British physicist John Henry Poynting.
- Poynting's theorem explains the energy density in electromagnetic waves.
- **Poynting Vector** represents the directional energy flux (the energy transfer per unit area per unit time) of an electromagnetic field. The SI unit of the Poynting vector is the watt per square meter (W/m^2). It is denoted by a letter P. $P = E \times H$.

Proof:

Let us consider an EM wave travelling along the x-axis with the magnetic and electric fields H and E confined in the transmission planes along z and y-axis respectively. This wave thus propagates along the direction of propagation vector $E \times H$.



$E \times H$ ------(1)

Taking divergence of equation (1)

$\nabla.(E \times H) = H.(\nabla \times E) - E.(\nabla \times H)$

We know that $\nabla \times E = -\frac{\partial B}{\partial t}$ and $\nabla \times H = \frac{\partial D}{\partial t}$ in free space

$\nabla.(E \times H = -H.\frac{\partial B}{\partial t} \; - \mathrm{E}.\frac{\partial D}{\partial t}$

$\nabla.(E \times H = -(\mathrm{E}.\frac{\partial D}{\partial t} + H.\frac{\partial B}{\partial t})$ -------(2)

We know that $D = \varepsilon_0$E and $B = \mu_0$H

Substituting these values in equation (2)

$\nabla.(E \times H = -(\varepsilon_0 \text{ E}.\frac{\partial E}{\partial t} + \mu_0 H.\frac{\partial H}{\partial t})$

$\nabla.(E \times H = -(\frac{1}{2}\varepsilon_0 \text{ 2E}.\frac{\partial E}{\partial t} + \frac{1}{2}\ \mu_0 2H.\frac{\partial H}{\partial t})$

$\nabla.(E \times H = -(\frac{1}{2}\varepsilon_0 \frac{\partial (E)^2}{\partial t} + \frac{1}{2}\mu_0 \frac{\partial (H)^2}{\partial t})$

$\nabla.(E \times H = -\frac{\partial}{\partial t}(\frac{1}{2}\varepsilon_0\ E^2 + \frac{1}{2}\varepsilon_0\ H^2)$

Considering the surface 'S'bounds a volume V and integrating the above relation over the volume V ,we get

$$\int_v \nabla.(E \times H) = -\frac{\partial}{\partial t}\int_v (\frac{1}{2}\varepsilon_0\ E^2 + \frac{1}{2}\varepsilon_0\ H^2)\text{dv}$$

- $\int_v \nabla.(E \times H) = -\frac{\partial}{\partial t}\int_v (\frac{1}{2}\varepsilon_0\ E^2 + \frac{1}{2}\varepsilon_0\ H^2)\text{dv}$

Applying Divergence theorem to the above equation

$$\int_s (E \times H).ds = -\frac{\partial}{\partial t}\int_v (\frac{1}{2}\varepsilon_0\ E^2 + \frac{1}{2}\varepsilon_0\ H^2)\text{dv}$$

The R.H.S of the above equation is the sum of energies of electric field and magnetic fields. Hence it represents amount of energy transferred over the volume v in one sec.

The vector $E \times H$ =P represents the amount of field energy passing through a unit area of surface in unit time normal to the direction of flow of energy.

3.9 Propagation of EM waves in non-conducting medium

Let us assume that the medium is perfectly non-conducting ,homogeneous and isotropic having permeability(μ) and permittivity (ε).

$$B = \mu H \ and \ D = \varepsilon E$$

The electrical conductivity for a dielectric is zero i.e σ =0 (J= σ μE)

The Maxwell fourth and third equations can be written as

$\nabla \times B = \mu_0 (j + \varepsilon_0 \frac{\partial E}{\partial t})$ = $\nabla \times B =$ μ $\varepsilon \frac{\partial E}{\partial t}$ -------(1)

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{ -------(2)}$$

On differentiating equation (1) we get

$\frac{\partial}{\partial t}(\nabla \times B) = \frac{\partial}{\partial t}(\mu \, \varepsilon \frac{\partial E}{\partial t})$

$\nabla \times \frac{\partial B}{\partial t} = \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2}$ ----(3)

Substituting (2) in (3) we get

$\nabla \times (-\nabla \times E) = \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2}$

$-\nabla \times (\nabla \times E) = \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2}$----(4)

From the property of ∇ operator we have

$\nabla \times (\nabla \times E) = \nabla(\nabla . E) - \nabla^2 E$

$-\nabla(\nabla . E) + \nabla^2 E = \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2}$ ------(5)

We know that $\nabla . E$ =0 (since there are no free charge carriers)

$\nabla^2 E = \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2} \Longrightarrow \nabla^2 E - \mu \, \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$ ------(6)

Similarly for the vector field B we can write

$\nabla^2 B - \mu \, \varepsilon \frac{\partial^2 B}{\partial t^2} = 0$ ------(7)

Comparing equation (6) and (7) with S.H.M equation $\frac{d^2y}{dt^2}$ - μy =0 ,which has velocity of wave propagation as $\frac{1}{\sqrt{\mu}}$

Therefore the field vectors satisfy the wave equation in which the velocity of wave propagation $\boldsymbol{v = C = \frac{1}{\sqrt{\mu\varepsilon}}}$

3.10 Propagation of EM waves in non-conducting medium

Let us assume that the medium is homogeneous , isotropic conducting medium having permeability(μ), permittivity (ε) and conductivity (σ).

$B = \mu H\ ,\ D = \varepsilon E\ \ and\ J = \sigma E$

The Maxwell's equations can be written as

1. $\nabla . E = \frac{\rho}{\varepsilon}$ -------(1)

2. $\nabla . B$=0 ------(2)

3. $\nabla \times E = -\frac{\partial B}{\partial t}$ --------(3)

4. $\nabla \times H = j + \frac{\partial D}{\partial t}$ (or) $\nabla \times B = \mu(j + \varepsilon\frac{\partial E}{\partial t})$ --------(4)

Applying curl on both sides of Maxwell's third equation

$\nabla \times (\nabla \times E) = -\nabla \times (\frac{\partial B}{\partial t})$

$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times \mathrm{B})$

Substituting (4) in the above equation

$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times \mathrm{B}) = -\mu\frac{\partial}{\partial t}(j + \varepsilon\frac{\partial E}{\partial t})$

But $J = \sigma E \Longrightarrow \nabla \times (\nabla \times E) = -\mu\frac{\partial}{\partial t}(\sigma E + \varepsilon\frac{\partial E}{\partial t})$ ------(5)

$\nabla \times (\nabla \times E) = \nabla(\nabla . E) - \nabla^2 E$-----(6)

Substituting (6) in (5)

$\nabla(\nabla . E) - \nabla^2 E = -(\mu\,\sigma\frac{\partial E}{\partial t} + \mu\,\varepsilon\frac{\partial^2 E}{\partial t^2})$

$-\nabla^2 E$= - (μ σ $\frac{\partial E}{\partial t}$ + μ ε $\frac{\partial^2 E}{\partial t^2}$)

$\nabla^2 E$= (μ σ $\frac{\partial E}{\partial t}$ + μ ε $\frac{\partial^2 E}{\partial t^2}$) ------(7)

Similarly $\nabla^2 B$= (μ σ $\frac{\partial B}{\partial t}$ + μ ε $\frac{\partial^2 B}{\partial t^2}$) ------(8)

Let us consider the EM wave is propagating along Z-direction, therefore the electric field is in x-direction and magnetic field is in y-direction.

Rewriting eq (7) by replacing E with E_x

$\frac{\partial^2 E_x}{\partial z^2}$= μ σ $\frac{\partial E_x}{\partial t}$ + μ ε $\frac{\partial^2 E_x}{\partial t^2}$ $------(9)$

The solution for the above equation is

E_x= E_0 $e^{i\omega t}$ ------------ (10)

differentiating eq (10) $\frac{\partial E_x}{\partial t}$=(i ω)E_0 $e^{i\omega t}$= i ω E_x---(11)

$\frac{\partial^2 E_x}{\partial t^2}$= = =(i ω) (i ω)E_0 $e^{i\omega t}$= - ω^2 E_x ------ (12)

Substituting (11) and (12) in eq (9)

$\frac{\partial^2 E_x}{\partial z^2}$= μ σ (i ω E_x)+ μ ε(- ω^2 E_x)

$\frac{\partial^2 E_x}{\partial z^2}$= E_x (μ σ i ω - μ ε ω^2)

$\frac{\partial^2 E_x}{\partial z^2} - E_x$ (μ σ i ω - μ ε ω^2) =0

$\frac{\partial^2 E_x}{\partial z^2} -$ $K^2 E_x$ =0 ------ (13)

Where K^2 = - μ ε ω^2 $+$ μ σ i ω

The solution of eq (13) is E_x= E_0 e^{-kx} ------ (14)

K consists of both real and imaginary parts

$$k = k_+ + ik_-$$

After solving (14) we will get

$k_+ = \omega \sqrt{\frac{\varepsilon\mu}{2}} \; [\sqrt{1 + (\frac{\sigma}{\varepsilon\mu})} + 1]^{1/2}$

Velocity $v \; (or) C = \frac{\omega}{k_+}$

Velocity V = $\sqrt{\frac{\boldsymbol{\varepsilon\mu}}{\mathbf{2}}} \; [\sqrt{\mathbf{1} + (\frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon\mu}})} + \mathbf{1}]^{\mathbf{1/2}}$

- For a poor conductor $\sigma \ll \varepsilon\omega$ then $k_+ \cong \omega\sqrt{\varepsilon\mu}$ then v $\cong \sqrt{\varepsilon\mu}$
- For a good conductor $\sigma \gg \varepsilon\omega$ then $k_+ \cong \sqrt{\frac{\omega\sigma\mu}{2}}$ then v $\cong \sqrt{\frac{\sigma\mu}{2\omega}}$

Penetration or skin depth: It is a measure of how deep light or any electromagnetic radiation can **penetrate** into a material. It is defined as the **depth** at which the intensity of the radiation inside the material falls to 1/e (about 37%) of its original value at (or more properly, just beneath) the surface. It is denoted by a letter d.

$d = \frac{1}{k_+} = \sqrt{\frac{2}{\omega\sigma\mu}}$

For a poor conductor skin depth is frequency independent

For a good conductor skin depth is frequency dependent