G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY
(AUTONOMOUS)
KURNOOL

Department of Humanities and Sciences

MATHEMATICS-1
QUESTION BANK

UNIT-I
MATRICES

1. Define the rank of the matrix **(JNTUA JUNE 2012,13, 14)**

2. Reduce the matrix $A=\begin{bmatrix}-1 & -3 & 3 & -1\\ 1 & 1 & -1 & 0\\ 2 & -5 & 2 & -3\\ -1 & 1 & 0 & 1\end{bmatrix}$ into echelon form and hence find its rank **(JNTUA JUNE 2011)**

3. Define the rank of the matrix and find the rank of the matrix $A=\begin{bmatrix}2 & 1 & 3 & 5\\ 4 & 2 & 1 & 3\\ 8 & 4 & 7 & 13\\ 8 & 4 & -3 & -1\end{bmatrix}$ **(JNTUA MAY 2005,06. SEP 2008)**

4. Determine the rank of the matrix $A=\begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$ by reducing into echelon form.

5. Find the value of k such that the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2 **(JNTUA 2006)**

6. Find the rank of $\begin{bmatrix}2 & -4 & 3 & -1 & 0\\ 1 & -2 & -1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 4 & -7 & 4 & -4 & 5\end{bmatrix}$ by reducing into echelon form. **(JNTUA 2008,09)**

7. Solve the following non- homogenous system of linear equations (AX =B)

i) Find the following system of equations are consistent if so solve them
$x+2y+2z=2,\ 3x-2y-z=5,\ 2x-5y+3z=-4,\ x+4y+6z=0$
(JNTUA 2001,02,04,05)

ii) Find whether the following equations are consistent, if solve them .
$x+y+2z=4;\ 2x-y+3z=9; 3x-y-z=2$ **(JNTUA MAY 2005)**

iii) Find whether the following set of equations are consistent if so, solve them.$x_1+x_2+x_3+x_4=0,$
$x_1+x_2+x_3-x_4=4,.x_1+x_2-x_3+x_4=-4,.x_1-x_2+x_3+x_4=2$ **(JNTUA MAY 2005)**

iv) Prove that the following set of equations are consistent and solve them.
$3x+3y+2z=1; x+2y=4; 10y+3z=-2; 2x-3y-z=5$
(JNTUA MAY 2006,07,08(K),09(H),09(K),10)

8. Solve the following homogenous system of linear equations (AX =O)

i) Solve the system of equations $x+3y-2z=0; 2x-y+4z=0; x-11y+14z=0$

ii) Solve the system of equations $x+y-3z+2w=0;\quad 2x-y+2z-3w=0; 3x-2y+z-w=0$
$-4x+y-3z+w=0$

iii) Show that the only real number λ $f$or which for which the system
$x+2y+3z=\lambda x;\ 3x+y+2z=\lambda y;\ 2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them, when $\lambda=6$
(JNTUA 2005,06,08)

iv) Solve the system of equations $x+y+w=0;\ \ y+z=0;\ \ x+y+z+w=0;\ x+y+2z=0$
(JNTUA 2008,09)

9. Find Eigen values and Eigen vectors of the following matrices

i)$A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$ **(JNTUA MAY 2006, 08,12)**

ii) $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ **(JNTUA MAY 2006,08)**

iii) $A=\begin{bmatrix}1 & -6 & -4\\ 0 & 4 & 2\\ 0 & -6 & -3\end{bmatrix}$ **(JNTUA 2008,10)**

iv) Verify that the sum of eigen values is equal to trace of $'A'$ for the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find the corresponding eigen vectors **(JNTUA 2007)**

10. DIAGONALIZATION AND CALUCULATION OF POWERS OF A MATRIX

i) Diagonalizable the matrix $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ and find A^4 **(JNTUA 2006)**

ii) If A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ find A) A^8 B) A^4 **(JNTUA 2006)**

iii) Diagonalize the matrix A = $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ **(JNTUA 2004,09)**

11. CAYLEY-HAMILTON THEOREM AND ITS PROBLEMS

i) Verify Cayley-Hamilton theorem for the matrix A= $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ .Hence find A^{-1} **(JNTUA 2005,06)**

ii) Verify Cayley-Hamilton theorem and find the characteristic roots where A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ **(JNTUA 2009)**

iii) Verify Cayley-Hamilton theorem for the matrix A= $\begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$ .Hence find A^{-1} **(JNTUA 2009)**

iv) Show that the matrix A = $\begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation. Hence find A^{-1}

UNIT-II

QUADRATIC FORMS & MEAN VALUE THEOREMS

Quadratic Forms:

1.i)Define Quadratic form

ii) Define Index, Signature, Nature of Quadratic form

2.Find the symmetric matrix of the following Quadratic forms

i)$x_1{}^2 + 6x_1x_2 + 5x_2{}^2$

ii) $x^2 + 2y^2 + 3z^2 + 4xy + 5yz + 6zx$

iii)$x_1{}^2 + 2x_2{}^2 - 7x_3{}^2 - 4x_1x_2 + 8x_1x_3 + 5x_2x_3$

iii)$x_1{}^2 + 2x_2{}^2 - 7x_3{}^2 - 4x_1x_2 + 8x_1x_3 + 5x_2x_3$

iv) $x_1{}^2 + 2x_2{}^2 + 4x_2x_3 + x_3x_4$

3.Find the Quadratic form relating to to the following matrices

i) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ ii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ iii) $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}$ iv) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

4. Reduce the following Quadratic forms to Canonical form (or) normal form (or) sum of squares form by using Orthogonal Transformation $X = PY$ *where P is an orthogonal matrix* and give the matrix of transformation. And also find Rank, Index, Signature, and Nature.

i) $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$

ii) $3x_1{}^2 + 3x_2{}^2 + 3x_3{}^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$

iii) $x_1{}^2 + 3x_2{}^2 + 3x_3{}^2 - 2x_2x_3$

iv) $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz$

v) $5x^2 + 26y^2 + 10z^2 + 6xy + 4yz + 14zx$

vi) $3x_1{}^2 + 5x_2{}^2 + 3x_3{}^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$

vii) $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2xz$

viii) $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$

Rolle's theorem:

5. Verify Rolle's Theorem for the following functions to the given intervals

i) $f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{2}, \sqrt{2}]$

ii) $f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{3}, \sqrt{3}]$

iii) $f(x) = \tan x$ in $[0,\pi]$

iv) $f(x) = \frac{1}{x^2}$ in $[-1, 1]$

v) $f(x) = x^3$ in $[1,3]$

vi) $f(x) = (x - a)^m (x - b)^n$ where m, n are positive integers in $[a, b]$

vii) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$

viii) $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a, b]$, $a > 0$, $b > 0$

ix) $f(x) = e^x Sinx$ in $[0, \pi]$

x) $f(x) = x(x+3)e^{-x/2}$ *in* $[-3,0]$

Legrange's Mean Value Theorem:

6. Verify LMVT for the following functions to the given intervals

i) $f(x) = \log_e x$ in $[1, e]$

ii)$f(x) = x^3 - x^2 - 5x + 3$ *in* $[0,4]$

iii) $f(x) = Cosx$ *in* $[0, \frac{\pi}{2}]$

iv) $f(x) = (x-1)(x-2)(x-3)$ in $[0,4]$

v) If $a < b$, prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ Using Lagrange's Mean value theorem .Deduce the following :

i) $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$

ii) $\frac{5\pi+4}{20} < tan^{-1}2 < \frac{\pi+2}{4}$

vi) Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < Sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$

TAYLOR'S SERIES

1. Find Taylor's series expansion of the following functions at the given point

i) $\sin x$ in powers of $x - \frac{\pi}{4}$
ii) e^x about $x = -1$
iii) $\sin 2x$ about $x = \pi/4$

2. Verify Taylor's theorem for $f(x) = (1-x)^{5/2}$ with lagranges form of remainder upto 2 terms in the Interval [0,1]

MACLAURIN'S SERIES

2. Find Maclaurin's Series expansion of the following functions

1) e^x
2) $\cos x$
3) $\sin x$
4) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$
5) Expand $\log_e x$ in powers $(x-1)$ and hence evaluate log 1.1 correct to 4 decimals.

UNIT-III

MULTIVARIABLE CALCULUS

1. Find first and second order partial derivatives of ax^2+2hxy+b y^2 and verify $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$.

2. Find first and second order partial derivatives of x^3+y^3-3axy and verify $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$.

3. If U= $\frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2\neq 0$ then prove that, $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}$ =0.

4. If U= $\log\left(x^3+y^3-3xyz\right)$, prove that, $\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right]^2 U=\frac{-9}{(x+y+z)}$.

5. If $x^2y^2z^2=e$ show that $x=y=z, \frac{\partial^2 z}{\partial x\partial y}=-(x\log ex)^{-1}$.

6. If $U=\log\left(x^2+y^2+z^2\right)$, prove that, $(x^2+y^2+z^2)\left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right]=2$.

7. If u= $\tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$, prove that, $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

8. If z= $\log(e^x+e^y)$ show that $rt-s^2=0$ where, $r=\frac{\partial^2 z}{\partial x^2}, t=\frac{\partial^2 z}{\partial y^2}, s=\frac{\partial^2 z}{\partial x\partial y}$.

9. If u= e^{xyz} , show that, $\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$.

10. Find first and second order partial derivatives of $\log(x^2+y^2)$.

11. Find the value of $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2}+\frac{1}{b^2}\frac{\partial^2 z}{\partial y^2}$ when, $a^2x^2+b^2y^2-c^2z^2=0$.

12. If u= $f(2x-3x,3y-4z,4z-2x)$ then prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$.

13. If u= $f(y-z, z-x, x-y)$ show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ (or) if u= $f(x-y, y-z, z-x)$, find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

14. If u= $f(r,s,t)$ where $r=\frac{x}{y}, s=\frac{y}{z}$ & $t=\frac{z}{x}$ show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

15. If $x=r\cos\theta, y=r\sin\theta$, then prove that, $\frac{\partial r}{\partial x}=\frac{\partial x}{\partial r}$ and $\frac{1}{r}\frac{\partial x}{\partial\theta}=r\frac{\partial\theta}{\partial x}$.

16. If u= $x\log xy$ where $x^3+y^3+3xy=1$ find, $\frac{du}{dx}$.

17. If $z=f(x,y)$ where $x=e^u+e^{-v}, y=e^{-u}-e^v$ show that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$.

18. Find $\frac{du}{dx}$ if

$(i)\ u=\sin(x^2+y^2)$ $(ii) u=\cos(x^2+y^2)$ where, $a^2x^2+b^2y^2=c^2$

JACOBIAN

1) If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$
2) If $x+y+z=u, y+z=uv, z=uvw$ evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$
3) If $x=r\cos\theta, y=r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$
4) If $x=u(1-v); y=uv$ Prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)}\times\frac{\partial(x,y,z)}{\partial(u,v,w)}=1$
5) If $u=x^2-y^2, v=2xy$ where $x=r\cos\theta, y=r\sin\theta$ show that $\frac{\partial(u,v)}{\partial(r,\theta)}=4r^3$
6) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$
7) If $x=a\cosh\theta\cos\emptyset, y=a\sinh\theta\sin\emptyset$, show that $\frac{\partial(x,y)}{\partial(\theta,\emptyset)}=\frac{1}{2}a^2(\cosh 2\theta-\cos 2\emptyset)$

Maxima & Minima of Functions of Two Variables

- Find Maxima& Minima Of the following functions

  i) $xy+\frac{a^3}{x}+\frac{a^3}{y}$
  ii) $\sin x+\sin y+\sin(x+y)$
  iii) $x^3y^2(1-x-y)$
  iv) $x^2+xy+y^2+\frac{1}{x}+\frac{1}{y}$
  v) $\sin x\sin y\sin(x+y)$ Where 0<x<π, 0<y< π
  vi) x^3+y^3-3axy
  vii) $x^3+3xy^2-3x^2-3y^2+4$
  viii) $x^4+y^4-2x^2+4xy-2y^2$
  ix) $x^3+3xy^2-15x^2-15y^2+72x$

Lagranges Multipliers with Three Varibles

1. Find minimum of $x^2+y^2+z^2$, given that $xyz=a^3$
2. Find minimum of $x^2+y^2+z^2$, given that $x+y+z=3a$
3. Find the volume of largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
4. Find a point on the plane $3x+2y+z-12=0$, which is nearest to the origin.
5. Find maximum of $u=x^2y^3z^4$, if $2x+3y+4z=a$
6. Find minimum of $x^my^nz^p$ if $x+y+z=a$
7. Find a point on the plane $3x+2y+z-12=0$, which is nearest to the origin.
8. Find the point on the plane $x+2y+3z=4$ that is closest to the origin

UNIT-IV

DOUBLE INTEGRALS

1. Evaluate the following integrals

i) $\int_0^3\int_1^2 xy(1+x+y)dydx$

ii) $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dxdy$

iii) $\int_0^1\int_0^1\frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$

iv) $\int_0^1\int_0^{\sqrt{(1+x^2)}}\frac{dydx}{1+x^2+y^2}$

v) $\int_0^2\int_0^x e^{x+y}dydx$

vi) $\int_0^5\int_0^{x^2} x(x^2+y^2)dxdy$

vii) $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$

viii) $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$

ix) $\int_0^{\infty}\int_0^{\pi/2} e^{-r^2}\,r\,d\theta\,dr$

x) $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta\;dr\,d\theta$

2. Evaluate $\iint_R y^2dxdy$ Where R is the region bounded by parabolas $y^2=4ax$ & $x^2=4ay$

3. Evaluate $\iint_R xydxdy$ Where R is the Region bounded by X-axis, ordinate $x=2a$ and the curve $x^2=4ay$

4. Evaluate $\iint(x+y)^2dxdy$ Where R is the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

5. Evaluate $\iint r\sin\theta\,dr\,d\theta$ over the cardioid $r=a(1-\cos\theta)$ above the initial line.

6. Show that $\iint r^2\sin\theta\,dr\,d\theta=\frac{2a^2}{3}$ over the region of semi circle $r=2a\cos\theta$ above the initial line.

9. Evaluate $\iint r^3dr\,d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$

10. Evaluate $\int_0^{\infty}\int_0^{\infty}e^{-(x^2+y^2)}\,dxdy$ by changing the polar co-ordinates. Hence show that $\int_0^{\infty}e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$

11. Evaluate the double integral $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)dy\,dx$

12. Transform the following to Cartesian form and hence Evaluate $\int_0^{\pi}\int_0^a r^3\sin\theta\cos\theta\,dr\,d\theta$

13. By changing into polar coordinates, evaluate $\iint\frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ (b>a).

14. Evaluate the integral by changing the order of integration $\int_0^{\infty}\int_0^{\infty}\frac{e^{-y}}{y}dydx$

15. Change the order of integration evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}}\,dydx$

16. Change the order of integration evaluate $\int_0^a\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dxdy$

17.Change the order of integration in integration $\int_0^1\int_{x^2}^{2-x}xy\,dxdy$ and hence evaluate the double integral

18.By changing the order of integration, evaluate $\int_0^3\int_1^{\sqrt{4-y}}(x+y)dxdy$

19.Change the order of integration solve $\int_0^a\int_{x^2/a}^{2a-x}xy^2dydx$

UNIT-V
TRIPLE INTEGRALS & SPECIAL FUNCTIONS

1. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dxdydz$
2. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$
3. Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)dx\,dy\,dz$
4. Evaluate $\int_1^{e}\int_1^{\log y}\int_1^{e^x} \log z\,dx\,dz\,dy$
5. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$
6. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}dz\,dy\,dx$
7. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$
8. Evaluate $\iiint xy^2z\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$
9. Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$
10. Evaluate $\int_0^1\int_0^{1-z}\int_0^{1-y-z} xyz\,dx\,dy\,dz$
11. Evaluate $\iiint(xy+yz+zx)dx\,dy\,dz$ where V is the region of space bounded by x=0, x=1, y=0, y=2, z=0, z=3

SPECIAL FUNCTIONS:

1. Symmetry of beta function, $i.e, B(m,n)=B(n,m)$.
2. Show that $B(m,n)=2\int_0^{\Pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$.
3. Show that $B(m,n)=B(m+1,n)+B(m,n+1)$.
4. If m and n are positive integers, then $B(m,n)=\frac{(m-1)!(n-1)!}{(m+n-1)}$.
5. Show that $B(m,n)=\int_0^{\infty}\frac{x^{m-1}}{(1+x)^{m+n}}dx=\int_0^{\infty}\frac{x^{n-1}}{(1+x)^{m+n}}dx$.
6. Show that $\int_0^{\Pi/2}\sin^m\theta\cos^n\theta\,d\theta=\frac{1}{2}B\left[\frac{m+1}{2},\frac{n+1}{2}\right]$.
7. Express the following integrals in terms of Beta function.

   $(i)\int_0^1\frac{x}{\sqrt{1-x^2}}dx$. $\qquad (ii)\int_0^3\frac{dx}{\sqrt{9-x^2}}$.

8. Prove that $\int_0^a(a-x)^{m-1}\ x^{n-1}dx=a^{m=n-1}\ B(m,n)$.

9. Show that $(i)\int_0^{\infty}\frac{x^{m-1}-x^{n-1}}{(1+x)^{m=n}}dx=0$. $\quad (ii)\int_0^{\infty}\frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=2B$.

10. Prove that $\int_0^1\frac{xdx}{\sqrt{1-x^5}}=\frac{1}{5}B\left[\frac{2}{5},\frac{1}{2}\right]$.

11. Show that $\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} B(m+1, n+1)$.

12. Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ in terms of Beta function.

13. Show that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ *where* $m > 0, n > 0$.

14. Prove that $\Gamma(n)\Gamma(1-n) = \frac{\Pi}{\sin n\Pi}$.

15. Find $\beta(2.5, 1.5)$.

16. Compute $\Gamma(4,5)$.

17. Find values of the following:

$(i)\Gamma\left(\frac{11}{2}\right)$ $\quad (ii)\Gamma\left[-\frac{1}{2}\right]$ $\quad (iii)\Gamma\left[\frac{5}{2}\right]$ $\quad (iv)\Gamma\left[-\frac{7}{2}\right]$ $\quad (v)\Gamma(10)$

18. Show that

$(i) B(m+1,n) = \frac{m}{m+n} B(m,n) \quad (m > 0, n > 0)$.

$(ii) B(m,n+1) = \frac{n}{m+n} B(m,n) \quad (m > 0, n > 0)$

$(iii) B(m,n) = B(m+1,n) + B(m,n+1) \quad (m > 0, n > 0)$ $\quad (iv)\beta\left[\frac{1}{2}, \frac{1}{2}\right] = \Pi$.

(v) Determine the value of $\beta(2,3)$.

19. Evaluate the following:

$(i)\int_0^1 x^5 (1-x)^3 \, dx$. $\quad (ii)\int_0^1 x^4 (1-x)^2 \, dx$ $\quad (iii)\int_0^2 x(8-x^3)^{1/3} \, dx$ $\quad (iv)\int_0^1 x^{5/2} (1-x^2)^{3/2} \, dx$.

20. Compute the following:

$(i)\int_0^\infty e^{-x} x^3 dx$. $\quad (ii)\int_0^\infty x^6 e^{-2x} dx$. $\quad (iii)\int_0^\infty e^{-4x} x^{3/2} dx$.

21. Evaluate the following:

(i) $\int_0^\infty x^2 e^{-x^2} dx$. (ii) $\int_0^\infty \sqrt{x} e^{-x^2} dx$. (iii) $\int_0^\infty e^{-a^2x^2} dx$ (iv) $\int_0^\infty e^{-x^2} dx$. (v) $\int_0^\infty e^{-\sqrt{x}} dx$.

22. Show that $\Gamma(n) = \int_0^1 \left[\log \frac{1}{x}\right]^{n-1} dx, n > 0$.

23.Evaluate the following:

(i) $\int_0^{\Pi/2} \sin^5\theta \cos^{7/2}\theta \, d\theta$. (ii) $\int_0^{\Pi/2} \sin^7\theta \, d\theta$. (iii) $\int_0^{\Pi/2} \cos^{11}\theta \, d\theta$.

(iv) $\int_0^{\Pi/2} \sqrt{\cos\theta} d\theta$. (v) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

24. Evaluate the following:

(i) $\int_0^\infty 3^{-4x^2} dx$. (ii) $\int_0^\infty a^{-bx^2} dx$. (iii) $\int_0^1 x^4 \left[\log \frac{1}{x}\right]^3 dx$. (iv) $\int_0^1 x^2 \left[\log \frac{1}{x}\right]^3 dx$.

25. Prove that $\int_0^{\Pi/2} \sin^2\theta \cos^4\theta d\theta = \frac{\Pi}{32}$.

26. Evaluate $\int_0^1 x^7 (1-x)^5 dx$ by using β,Γ functions.

27. Evaluate the following:(by using β,Γ functions)

(i) $\int_0^{\Pi/2} \sin^{7/2}\theta \cos^{3/2}\theta d\theta$. (ii) $\int_o^{\Pi/2} \sin^6\theta \cos^7\theta \, d\theta$.

(iii) $\int_0^{\Pi/2} \sin^5\theta d\theta$. (iv) $\int_0^{\Pi/2} \sin^5\theta d\theta$.