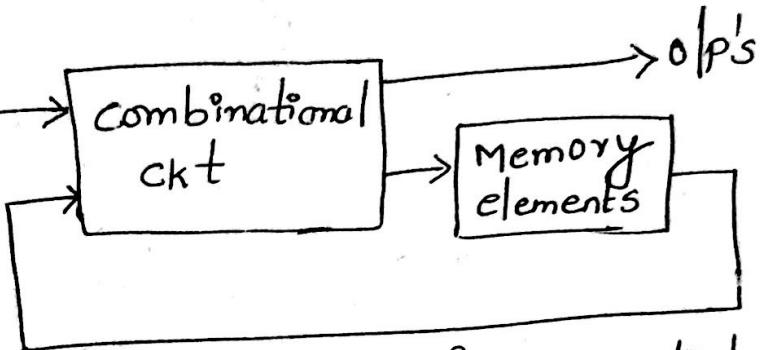


Analysis and Synthesis of Sequential Circuits:-

Sequential circuits:- → In case of sequential circuits the output will depends on the present input and as well as past output.

→ It consists of a combinational circuit to which storage elements are connected to form a feedback path.

→ i/p's



Block diagram of sequential circuit

→ The binary information stored in these elements at any given time defines the state of the sequential circuit at that time: present state.

→ Next state:- The present state that is fed back via memory element and external i/p's given to a combinational ckt will determines the output and this will become the next state of sequential circuit.

Present state Next state

A_n	B_n	C_n	A_{n+1}	B_{n+1}	C_{n+1}
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0

Differences between combinational and sequential circuits:-

→ The behavior of an asynchronous Sequential circuit depends upon the input signals at any instant of time and the order in which the inputs change. [storage elements: time delay devices]

Differences between Synchronous and Asynchronous

Synchronous

1. The i/p signal can affect the memory elements upon the activation of the clock pulse.
2. Memory elements: clocked flipflops.
3. Synchronization is employed with the help of clock pulse.
4. The maximum operating speed of clock pulses depends on the time delays (or) propagation delay of a circuit.

5. Easy to design

6. overcome instability : time is easily broken down into independent discrete so separately.

Synchronous clocked

- A Synch seq ckt uses signals that affect the storage elements at only discrete instants of time.

Asynchronous

1. The change in the i/p signal can affect memory element at any instant of time.

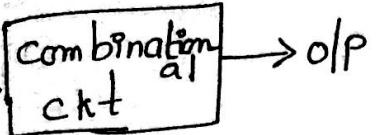
2. Memory elements: unclocked flipflops (or) time delay elements.

3. There is no synchronization, hence it is a combinational ckt with feed back.

4. Because of absence of clock pulse, asynchronous cks are operated faster than the synchronous ckt.

5. More difficult to

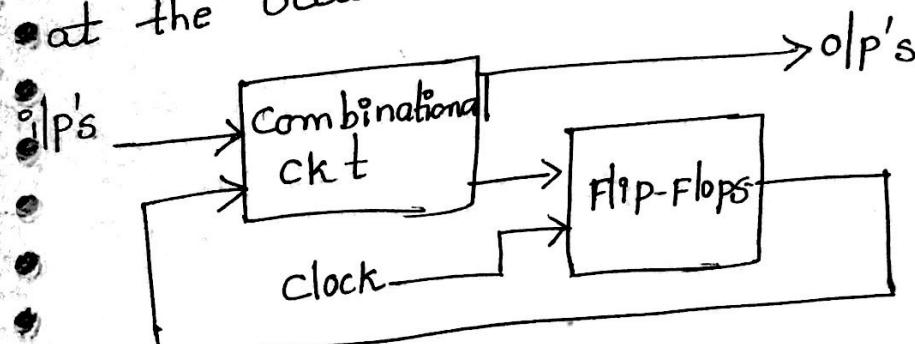
6. Because of feedback among logic gates, it becomes unstable at times.

- Combinational ckt
- The outputs will depends upon the present inputs
- 1. The outputs will depends upon the present input and past output.
- 2. $q/p \rightarrow$  $\rightarrow O/P$
- 3. Memory element is not present.
- 4. Combinational ckt's are easy to design but require more hardware.
- 5. Ckt's are faster in speed because there is no memory element.
- 6. More expensive ckt.
- Ex:- parallel adder
- Less flexibility o/p depends on Pilp only.
- 7. Due to the presence of memory element sequential circuit works with less speed.
- 8. cheaper
- 9. Ex:- Serial adder
- 10. More flexible o/p depends on Pilp & past o/p

Classification of sequential circuits:-

- Here classification of Sequential circuits is a function of the timing of their signals.
- 2 types.
- A Synchronous Sequential circuit, is a system whose behavior can be defined from the knowledge of its signals at discrete instances of time.

- Synchronization is achieved by a timing device called a clock generator, which provides a clock signal having the form of a periodic train of clock pulses. (clk)
- The clock pulses are distributed throughout the system in such a way that storage elements are affected only with the arrival of each pulse.
- For eg, a ckt that is to add and store two binary numbers would compute their sum from the values of the numbers and store the sum at the occurrence of a clock pulse.



(a) Block diagram

Asynch
ckt +
Logic gates
[feedback]



(b) Timing diagram of clock pulses.

0	0	0	NAND
0	1	1	
1	0	0	
1	1	1	
0	0	0	

Synchronous clocked sequential circuit:

- synchronous sequential circuits that use clock pulses to control storage elements are called as clocked sequential circuits.

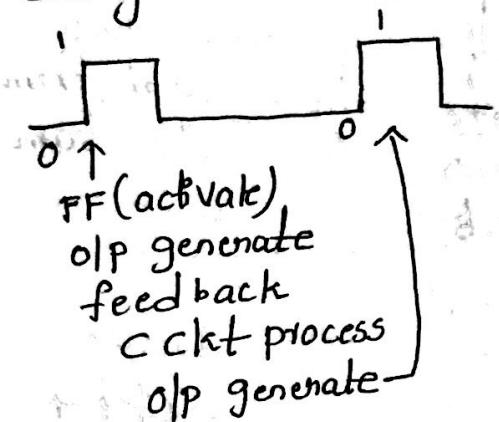
→ The storage elements (memory) used in clocked sequential circuits are called as flipflops.

→ A FF is a binary storage device capable

of storing one bit of information.

→ A sequential circuit may use many flip-flops to store as many bits as necessary.

→ A FF will not respond to a op's of combinational ckt until a clock pulse is changed from 0 to 1.



→ FF / Bistable multivibrator
2 states
Logic 0 & Logic 1.

Latches:- → A storage element in a digital circuit that can maintain a binary state as long as power is delivered to the circuit. indefinitely.

→ storage elements that operate with signal levels rather than signal transitions [↑ / ↓] one referred to as latches.

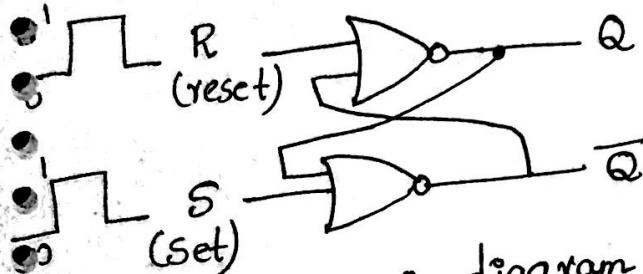
→ Latches are said to be level sensitive devices. FF's are edge-sensitive devices.

→ Latches are the basic circuits from which all flip-flops are constructed. Latches are used for the design of asynchronous sequential circuits.

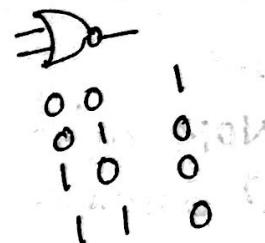
1) SR Latch:

→ The SR latch is a circuit with two cross-coupled NOR gates (or) two cross-coupled NAND

- gates as S = Set, R = Reset
- The Latch has two useful states. When $Q=1$, $\bar{Q}=0$
 - the latch is said to be in set state.
 - $O/P Q=0, \bar{Q}=1 \rightarrow$ reset state.
 - Q, \bar{Q} are complement of each other.
 - when both o/p's are 1 the o/p's are 0's known as unpredictable / undefined / metastable state.



(a) Logic diagram



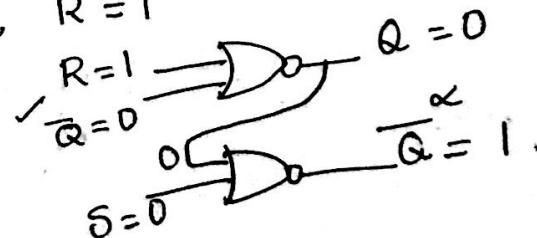
(a). Let $S=1, R=0$, (assume $\bar{Q}=0$)

$$R=0 \rightarrow Q=1 \Rightarrow Q=1.$$

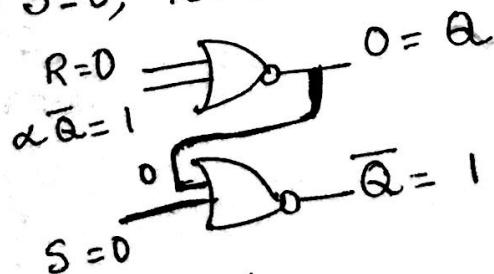
(b). $S=0, R=0$ $\bar{Q}=0 \rightarrow Q=1$ $Q=1 \rightarrow \bar{Q}=0$ (no change)

$=$ 1st step
 $=$ 2nd step P.

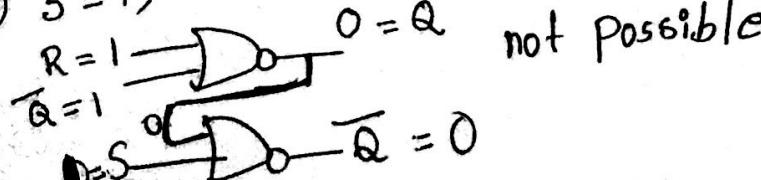
(c). $S=0, R=1$



(d). $S=0, R=0$



(e). $S=1, R=1$

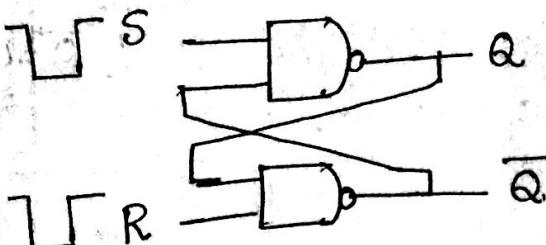


(b) Function table

S	R	Q	\bar{Q}
1	0	1	0
0	0	1	0 (No change)
0	1	0	1
0	0	0	1 (No change)
1	1	0	0 [Not Possible]

$\frac{\text{1st}}{\text{2nd}}$
undefined
forbidden

-IS of
R at



S R	Q	\bar{Q}
1 0	0	1
1 1	0	1 (after $S=1, R=0$)
0 1	1	0
1 1	1	0 (after $S=0, R=1$)
0 0	1	1 X

SR Latch with NAND

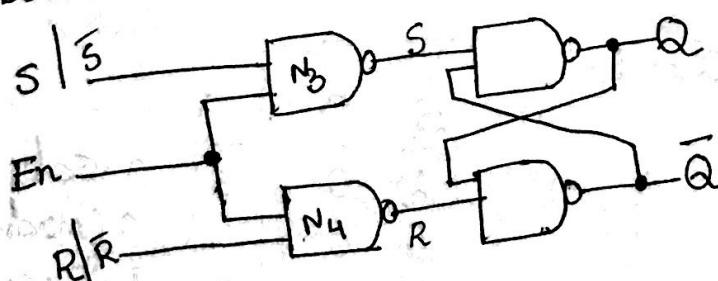
→ In comparing the NAND with the NOR Latch, note that the input signals for the NAND require the complement of those values used for the NOR latch.

→ Because the NAND latch requires a '0' signal to change its state, → SR Latch

→ The primes (bars over the letters) designate the fact that the inputs must be in their complement form to activate the circuit.

→ An SR latch with a control input is shown below. It consists of the basic SR latch and two additional NAND gates.

→ The two gates are called as steering | control gates because they are used to control the outputs



Logic diagram

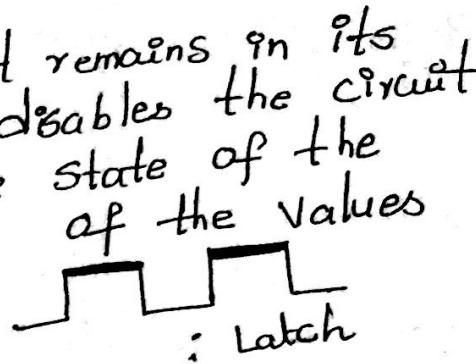
En	S	R	Next state of Q
0	x	x	NO change
1	0	0	NO change
1	0	1	$Q=0$; reset
1	1	0	$Q=1$; set
1	1	1	Indeterminate

→ The control input En acts as an enable signal for the other two inputs.

→ The outputs of the NAND gates stay at the

logic-1 level as long as the enable signal remains 1. 0. → quiescent condition for the SR latch. When the enable input goes to 1, information from the S or R input is allowed to affect the latch.

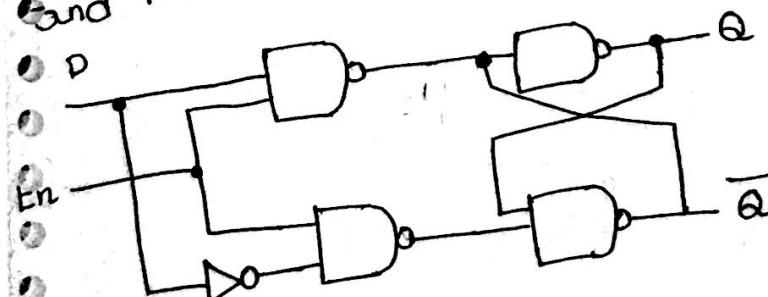
when E_n returns to 0, the circuit remains in its current state. The control input disables the circuit by applying 0 to E_n , so that the state of the output does not change regardless of the values of S and R.



D (Data) Latch (or) Transparent Latch :-

→ one way to eliminate the undesirable condition of the indeterminate state in the SR latch is to ensure that inputs S and R are never equal to 1 at the same time.

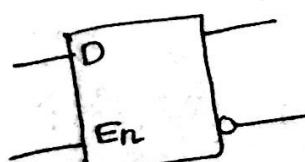
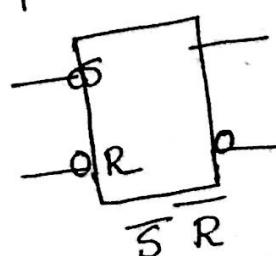
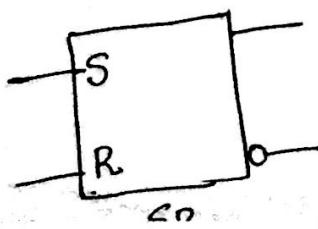
→ This is done in 'D' Latch, it has only two inputs D and E_n . The D input goes directly to the S input, and its complement is applied to the R input.



Logic diagram.

NAND also

→ Next state of $Q \mid Q_{n+1} = D$ is the characteristic eqn.

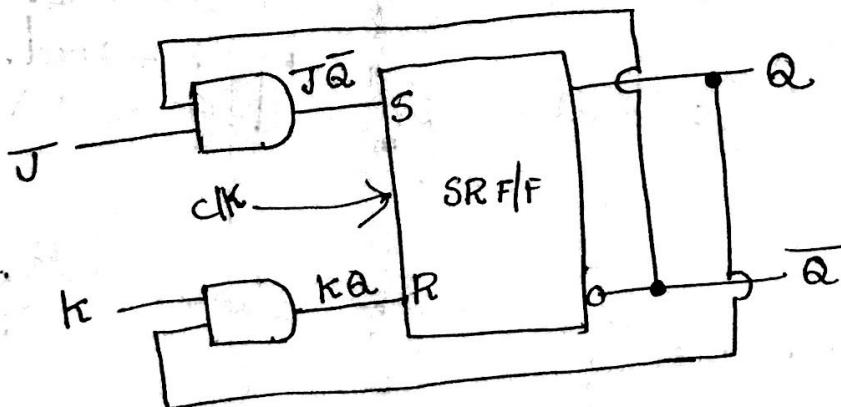


Graphic symbols

- The D latch receives that designation from its ability to hold data in its internal storage.
- It is used as a temporary storage for binary information between a unit and its environment, so it is called a transparent latch.

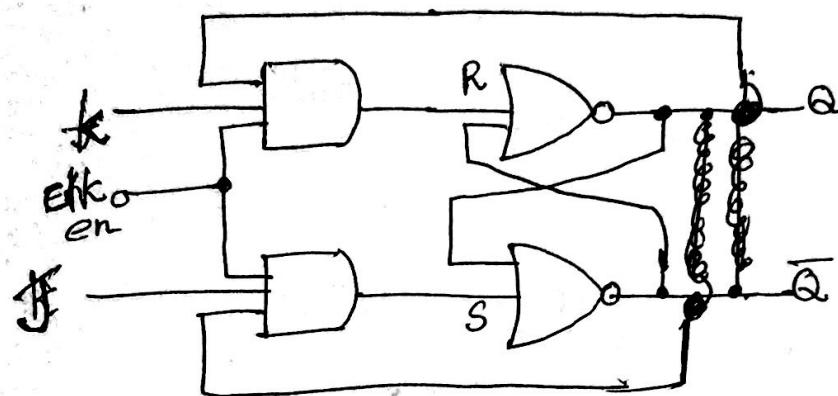
Flip-Flops:-

- Note :-
- 1) Latch responds only at positive levels.
 - 2) FF responds at edge triggerings.
- In order to overcome the indeterminate state in SR FF we change a little modification to a ckt which results in J-K FF.
- Here the input signal does not go to the S input of the SR FF directly but it is gated via an AND gate.
- The J input of the AND gate is one of the input lines of the new FF. The other i/p of the AND gate is the \bar{Q} output of the Flipflop. Similarly the new i/p line K is created via AND gate.



J-K FF circuit obtained by modifying the S/R FF circuit.

→ It can now be seen that the feedback to one of the AND gates is Q , whereas the feedback to the other AND gate is \bar{Q} . ($\therefore Q \& \bar{Q}$ are complement to each other) so, the two AND gates cannot produce the output 1 simultaneously.



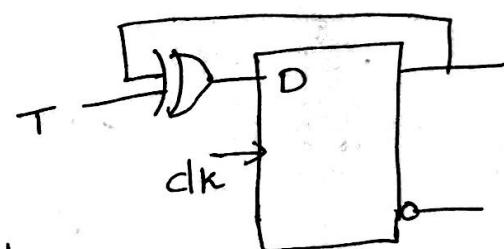
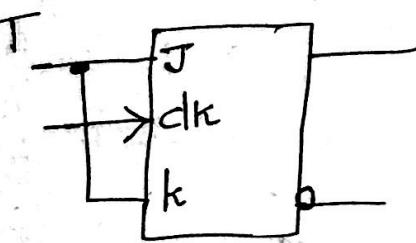
J	K	Q_{n+1}
0	0	No charge
0	1	0 Reset
1	0	1 Set
1	1	\bar{Q} comple

characteristic table.

T (Toggle F/F) :-

T F/F by D :-

Func	
J	K
0	0
0	0
0	1
0	0
1	0
1	1

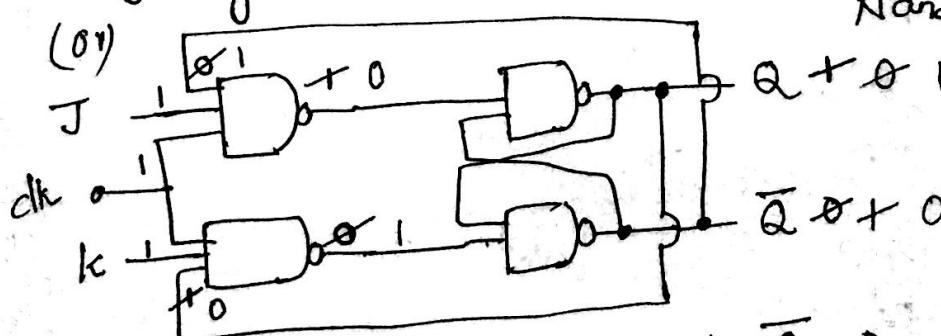


$$T=0, \bar{Q}_{n-1}$$

Later we discuss them in conversions.

→ Toggling = complementing.

Nand = complemented



1) $clk=1, J=1, K=0$, no 3rd o/p, $Q=1, \bar{Q}=0$

2) $clk=1, J=0, K=1, Q=0, \bar{Q}=1$

3) $clk=1, J=1, K=1$, assume $Q=1, \bar{Q}=0$

Flip Flop characteristic Tables :-

J	K	Q _{n+1}	R	S	Q _{n+1}
0	0	NO change	0	0	No charge
0	1	0 reset	0	1	1 set
1	0	1 set	1	0	0 reset
1	1	$\overline{Q_n}$ complement	1	1	indeterminate

D	Q _{n+1}	T	Q _{n+1}
0	0	0	D(t) No change
1	1	1	$\overline{D}(t)$ complement

Characteristic equations:-

$$1) Q_{n+1} = D \quad 2) Q_{n+1} = T \oplus Q_n$$

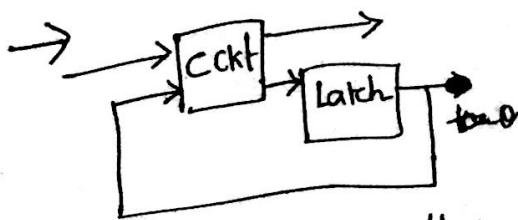
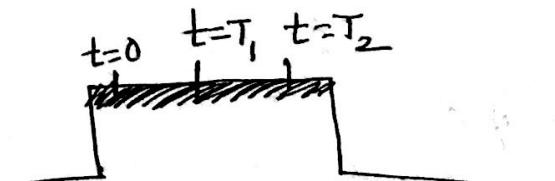
$$3) Q_{n+1} = \overline{J}Q_n + \overline{K}Q_n \quad 4) Q_{n+1} = S + \overline{R}Q_n$$

→ These eqn's are derived from characteristic tables and ckt dgm.

Race around condition:-
→ In an JK Latch ckt, the o/p's are responds only to positive levels.



Response to positive level.



- a) at $t=0$, the Latch takes i/p from CC & produces

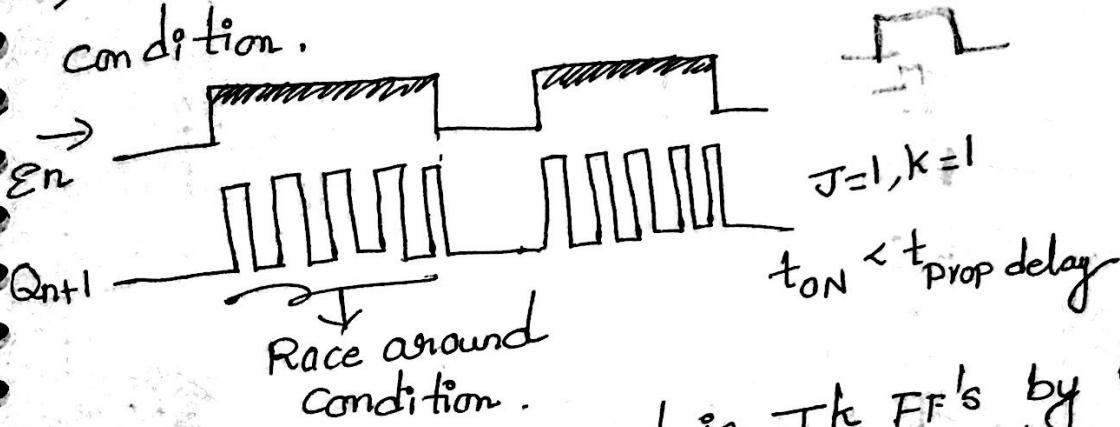
o/p and again given to Latch at $t = T_1$
 b) At $t = T_1$, again the Latch takes input from ckt & produces o/p & again given to Latch at $t = T_2$

→ Like this manner o/p's of Latch changes as

$$Q = 0 \rightarrow 1 \rightarrow 0 \rightarrow 1 \dots \quad \left. \begin{array}{l} \text{unreliable values} \\ \text{when } J=k=1. \end{array} \right\}$$

$$\bar{Q} = 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \dots$$

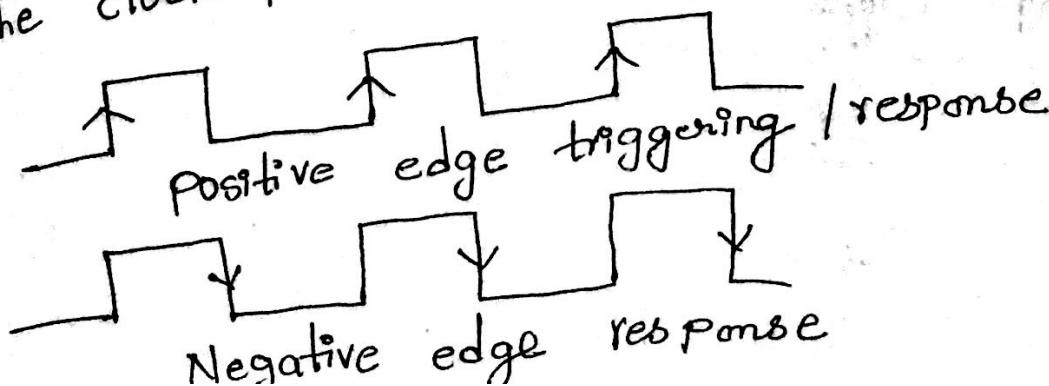
→ This is called as Toggling (or) race around condition.

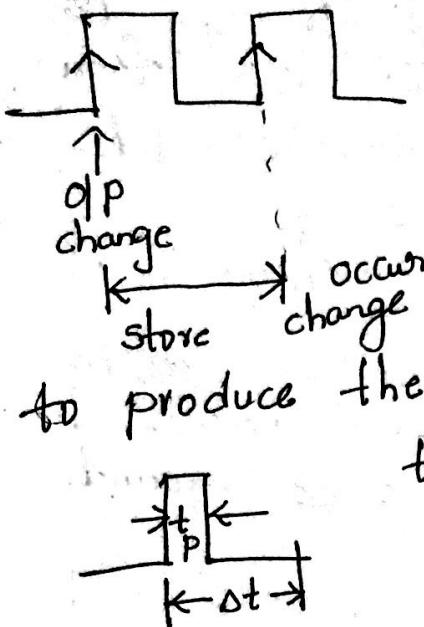


→ This can be overcome in JK FF's by using 3 ways:

1). To employ 2 latches, in a special configuration that separates the o/p of one FF and prevents it being affected while the i/p to the other FF is changing, ie, master-slave JK FF.

2). Another way is to produce a FF that triggers only during a signal transition (from 0 to 1 or from 1 to 0) and disabled during the rest of the clock pulse.





3) To maintain a condition
ie, pulse width \leq propagation delay time.

\rightarrow propagation time is defined as the time taken by the circuit to produce the output for a given inputs.

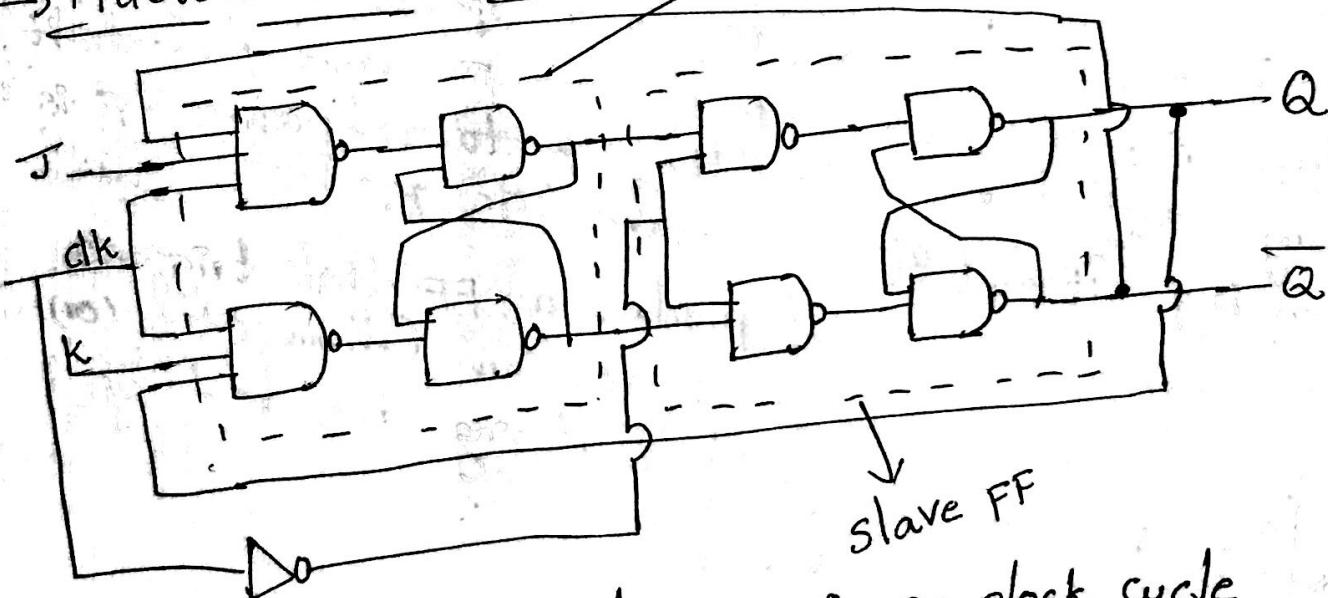
$$t_p \leq \Delta t$$

$$\text{clk} \quad t_p = \Delta t$$

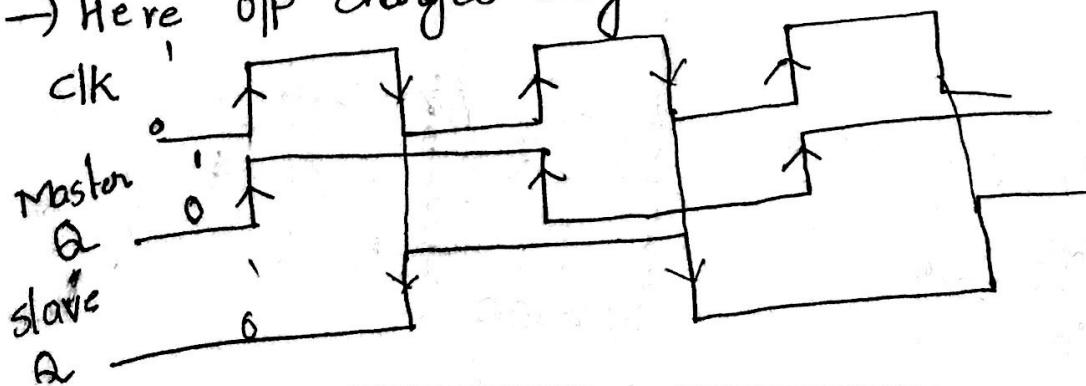
Q_{n+1} NO race around

\rightarrow But it can't be used practically :: Such a narrow pulse width cannot be generated (By \uparrow freq) but it cannot be used practically due to internal delays of a circuit.

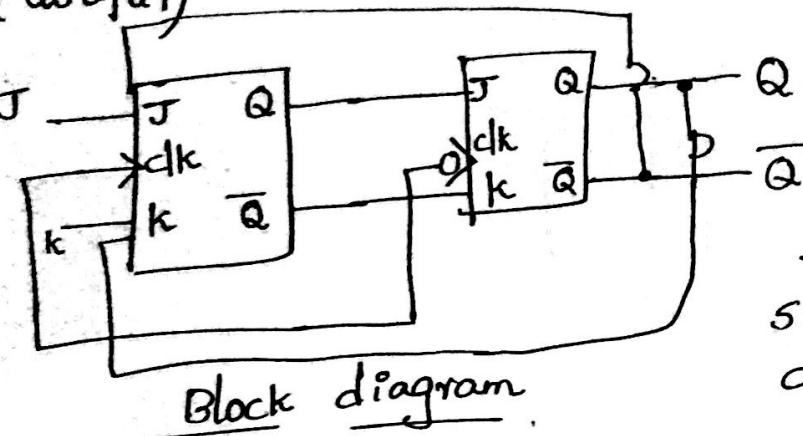
Master - slave Jk Flipflop :- master FF



\rightarrow Here o/p changes only once in a clock cycle



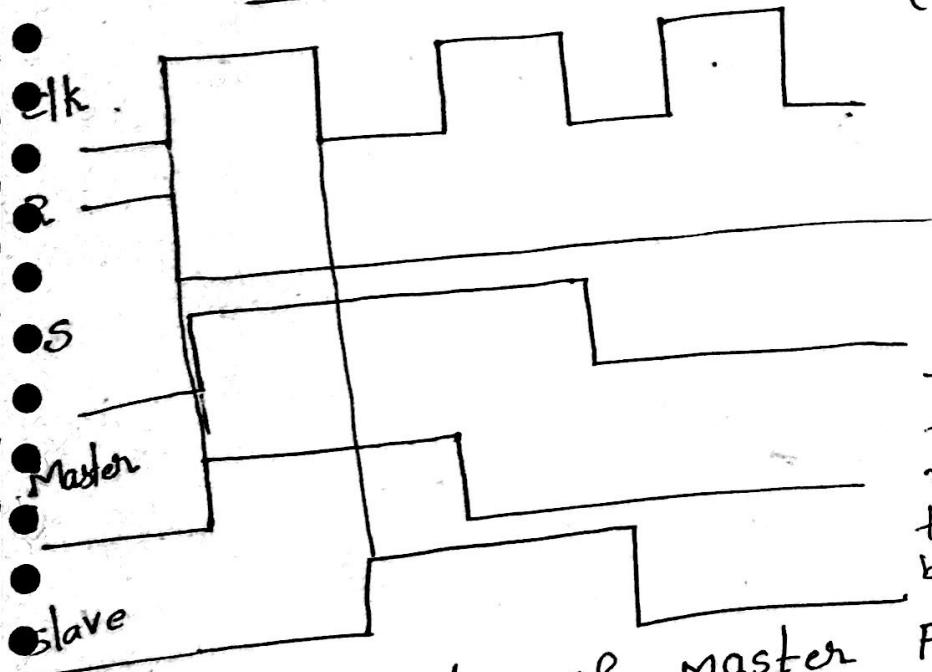
→ constant toggling is not there, toggling is a controllable one where as racing uncontrollable change (undesirable). (useful)



→ If $clk=1$, then slave $clk=0$, then we can't observe any o/p (\because Master o/p is in locking condition).

→ So, $clk=0$, i.e., FF changes o/p only once so eliminating 'racing'.

→ $J=K=1$, master toggles on the F. & slave copies the o/p of master on F. Now, feedback i/p to the master FF are complemented but as it is \bar{F} master FF inactive. grace around condition. FF is determined by



→ The o/p state of master J and K inputs at the +ve edge clock pulse. The output state of master is then transferred as an input to the slave FF.
→ The slave FF uses this input at -ve clock pulse to determine its output state.

Conversions of Flip Flops:-

→ In order to do conversions we need excitation tables:

J FF :-

(NS)		T
Q _n	Q _{n+1}	
0	0	0
0	1	1
1	0	1
1	1	0

$$Q_{n+1} = T \oplus Q_n$$

SR FF :-

Q _n		S	R
Q _n	Q _{n+1}		
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

D FF :-

Q _n	Q _{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

$$Q_{n+1} = D$$

JK FF :-

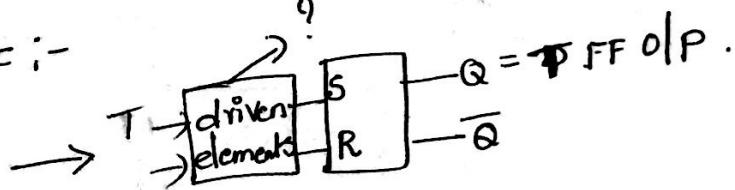
Q _n	Q _{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

→ Ext tables are obtained from characteristic eqn's:

$$X = 0, 1.$$

SR to T FF :-

↓ available FF



Input Q_n Q_{n+1}

FF 0/P's
S R

Q _n	0	1
0	0	X
1	1	0

Input	Q _n	Q _{n+1}	FF 0/P's
T			S R
0	0	0	0 X
1	0	1	1 0
1	1	0	0 1
0	1	1	X 0

3rd.

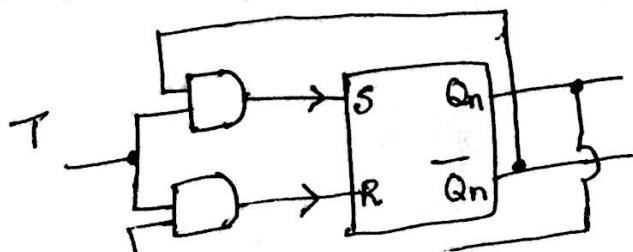
2nd 0st

Q _n	0	1
0	X	0
1	0	1

$$R = T \bar{Q}_n$$

Q _n	0	1
0	X	0
1	0	1

$$S = T \bar{Q}_n$$

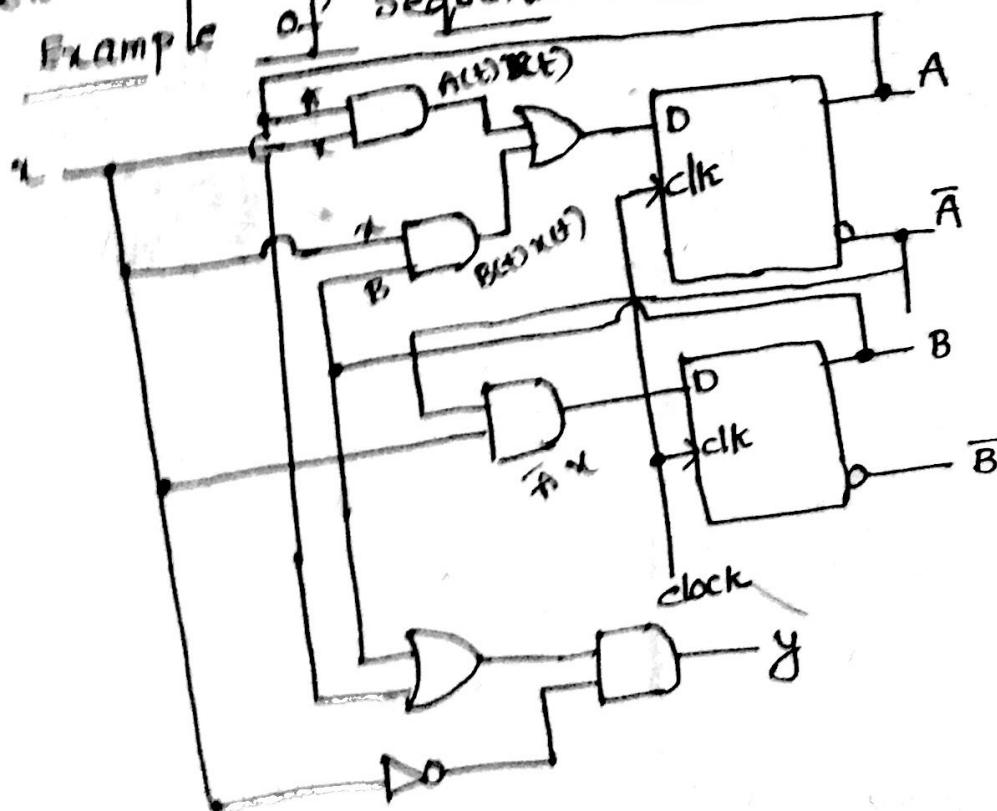


- 1) JK to D FF
- 2) T to D FF
- 3) SR to JK FF
- 4) JK to T FF

Analysis of clocked sequential circuits :-

- Analysis describes what a given circuit will do under certain operating conditions.
- The behavior of a clocked sequential circuit is determined from the inputs, outputs and the state of the flip-flops.
- The outputs and the next state are both a function of the inputs and the present state.
- The analysis of a sequential circuit consists of obtaining a table or a diagram.
- A logic diagram is recognized as a clocked sequential circuit if it includes flip-flops with clock inputs.

Example of Sequential circuit :-



$$\text{I/P's} = A, B, x \\ \text{O/P} = y$$

state equations :- → describes behavior of a clocked seq. ckt
 → It is also called as transition equation specifies the next state as a function of present state & inputs. $A(t) = P.S$, $A(t+1) = N.S$

$$i) A(t+1) = A(t)x(t) + B(t)\dot{x}(t)$$

$$\text{ii) } B(t+1) = \bar{A}(t) \cdot x(t)$$

$$iii) \quad y(t) = [B(t) + A(t)] \bar{x}(t)$$

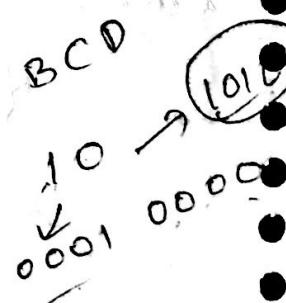
state Table :-

State Table :-

- The time sequence of inputs, outputs and flip-flop states can be enumerated in a state table. (transition)
- The table consists of PS, i/p's, NS & o/p.

state Table

State Table :-		input x	Next state A^+	B^+	output y	
Present state	A B				0	1
0 0		0	0	0	0	0
0 0		1	0	1	1	0
0 1		0	0	0	0	1
0 1		1	1	1	1	0
1 0		0	0	0	0	1
1 0		1	1	0	0	0
1 1		0	0	1	0	0
1 1		1	1	0	0	1



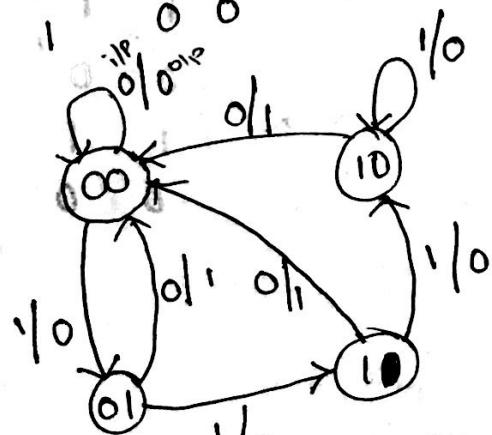
state state diagram :- → The information available in a table can be represented graphically in the form of a diagram.

Form of a state diagram.

- In this type of diagram, a state is represented by a circle, and the (clock triggered) transitions between states are indicated by directed lines connecting the circles.

Reduced form of state Table:-

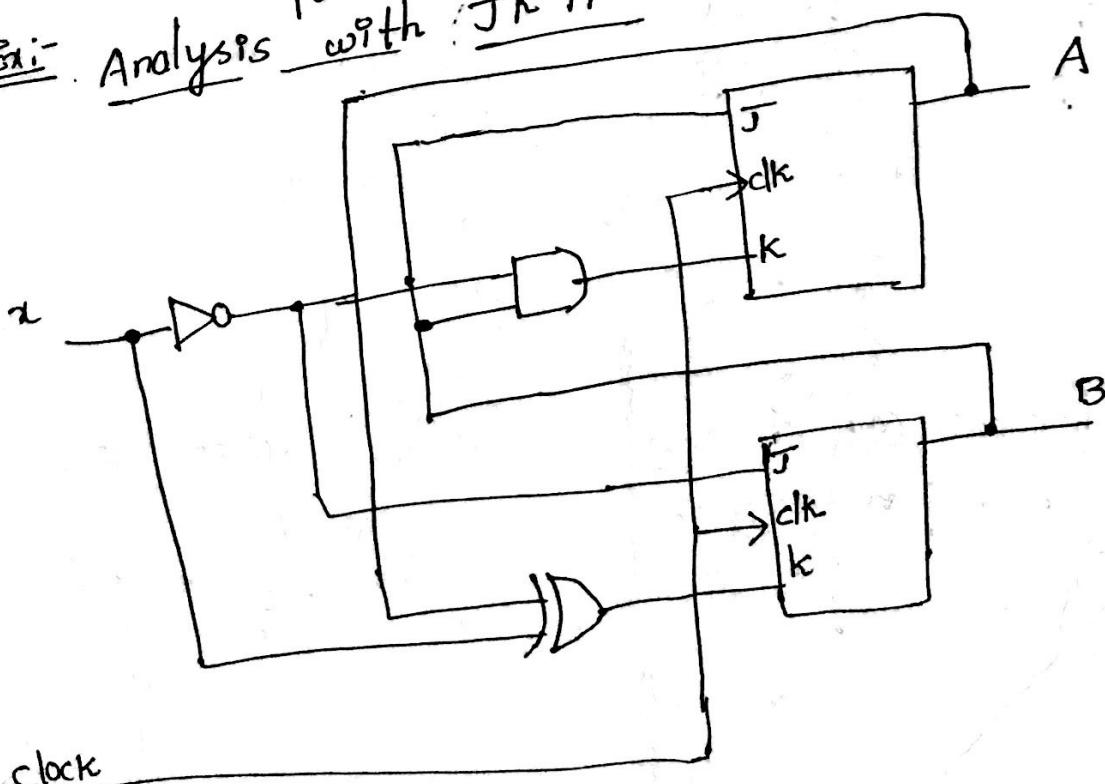
PS	NS		0/P	
	$x=0$	$x=1$	$x=0$	$x=1$
A B	A B	A B	y	y
0 0	0 0	0 1	0	0
0 1	0 0	1 1	1	0
1 0	0 0	1 0	1	0
1 1	0 0	1 0	1	0



state eqn's
→ State Table →
ckt dgm → State dgm.

modu - 12

Ex:- Analysis with Jk FF:-



1) State eqn's:-

$$AC(t+1) = \overline{J_A} + \overline{K_A}$$

58

$$i) \overline{J_A} = B, \quad k_A = \frac{B}{\overline{x}}$$

$$q) \quad \frac{A}{J_B} = \bar{x}, \quad k_B = \bar{A}x + A\bar{x} = A(\bar{x} + x) = A(t+1)^2 B \bar{A} + B \bar{x} A$$

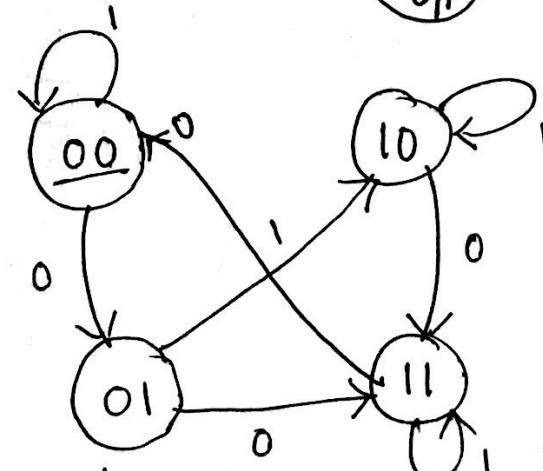
2) State Table :-

PS			S/P		NS		A ^t (t+1)		B ^t (t+1)		Flip Flop inputs		J _B		k _B	
A	B	x									J _A	k _A				
0	0	0	0		1						0	0	1	0		
0	0	1	0		0	0					0	0	0	1		
0	1	0	1		1	1					1	1	1	0		
0	1	1	1		0						1	0	0	1		
0	0	0	1		1	1					0	0	1	1		
1	0	1	1		0	0					0	0	0	0		
1	1	0	0		0	0					1	1	1	1		
1	1	1	1		1	1					1	0	0	0		
①st			3rd				2nd									

1st

J	k	Q_{n+1}
0	0	Q_n
0	1	0
-1	0	<u>1</u>
-1	1	<u>Q_n</u>

3) state dgm:

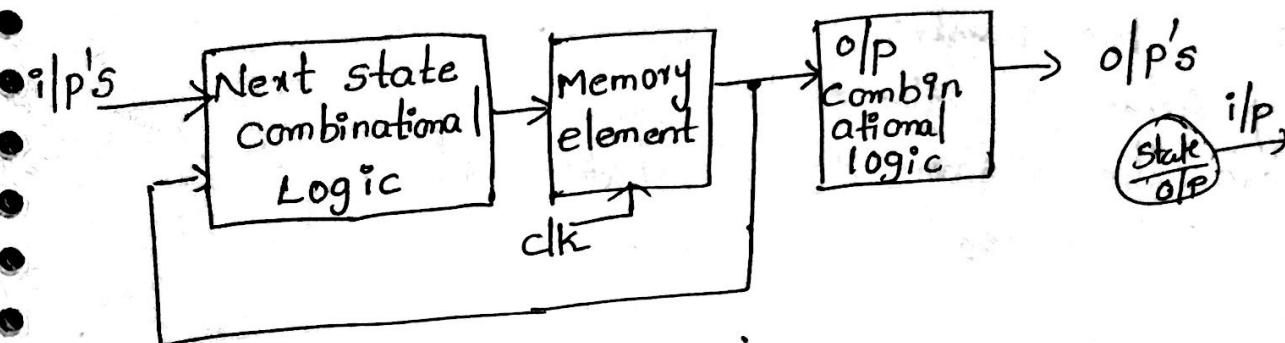


→ The Synchronous (or) clocked sequential ckts are represented by two models:

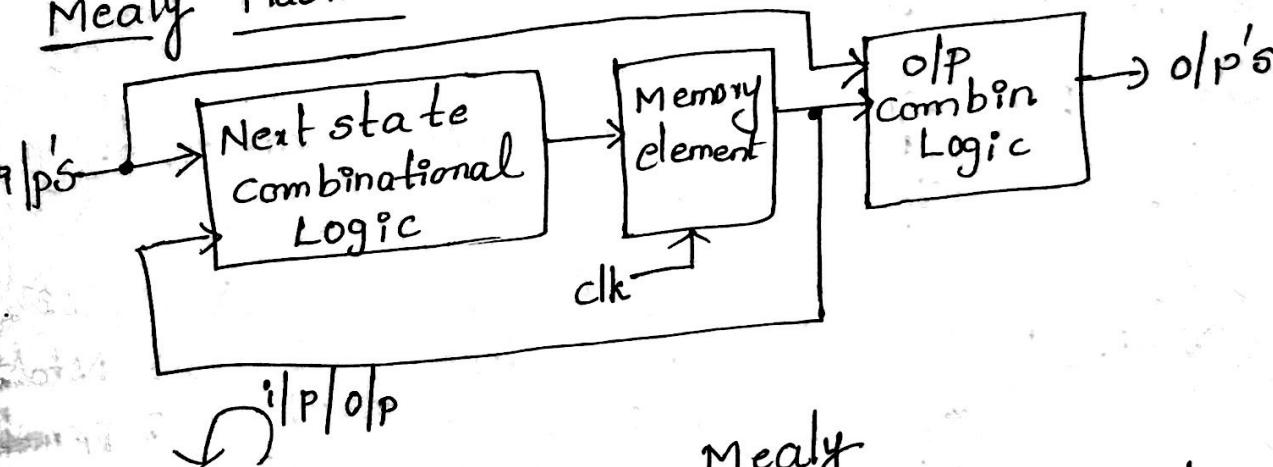
v) Moore Model:- The output depends only on the present states of the FF.

Q) Mealy ckt :- The output depends on both the present state of the FF's and on the inputs.

Moore machine :-



Mealy Machine :-



Moore

→ Its output is a function of present state only

→ I/P changes doesn't affect the output.

→ This circuit requires more no. of states for implementing the functions

→ $Z(t) = f\{s(t)\}$
i/p

Mealy

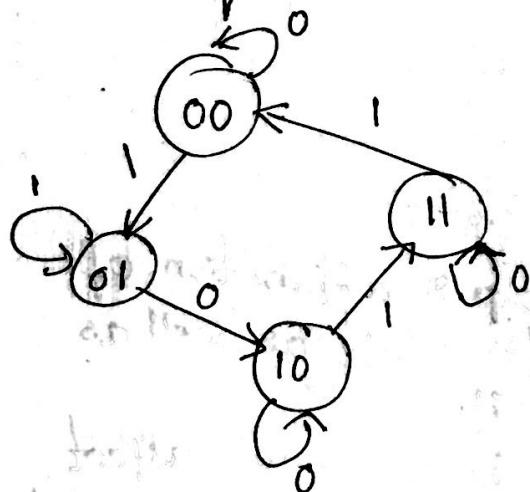
- Its o/p is a function of present state as well as present i/p.
- I/P changes may affect the output of the circuit.
- Less no. of states

$$\rightarrow Z(t) = f\{s(t), x(t)\}$$

~~synchronous sequential ckt~~
sign & procedure :-

- Steps:-
- 1) From the word description and specifications of the desired operation, derive a state diagram for the circuit.
 - 2) Reduce the number of states if necessary & Assign binary values to the states
 - 3) Obtain the binary coded state table
 - 4) choose the type of flip-flops to be used.
 - 5) Derive the Simplified FF I/P eqn's & O/P eqn's
 - 6) Draw the logic diagram.

Example 1 :- Given the following state diagram, design the Sequential circuit using JK flip-flops.



State Table

PS	NS	2^m States
A B	X = 0 $A^+ B^+$	$2^2 = 4$ States
	X = 1 $A^+ B^+$	2 = FF needed
00	00	01
01	10	01
10	10	11
11	00	00

JK FF Excitation Table

Q	Q ⁺	J	K
0	0	0	X
0	1	X	
1	0		X
1	1	X	0

PS	I/P	NS	FF I/P's	
A B	X	$A^+ B^+$	$\bar{J} A$	K_A
0 0	0 0	0 0	0	X
0 1	0 0	0 1	0	X
1 0	0 1	1 0	1	X
1 1	0 1	0 1	0	X
	1 0	1 0	X 0	0 X
	1 0	1 1	X 0	1 X
	1 1	0 1	X 0	X 0
	1 1	0 0	X 1	X 1

K-map :-

(61)

		J	A			
		00	01	11	10	
A		0	0	1	X	X
1		X	X	X	X	

$$J_A = B\bar{X}$$

		J	B		
		00	01	11	10
A		0	1	X	X
1		1	X	X	

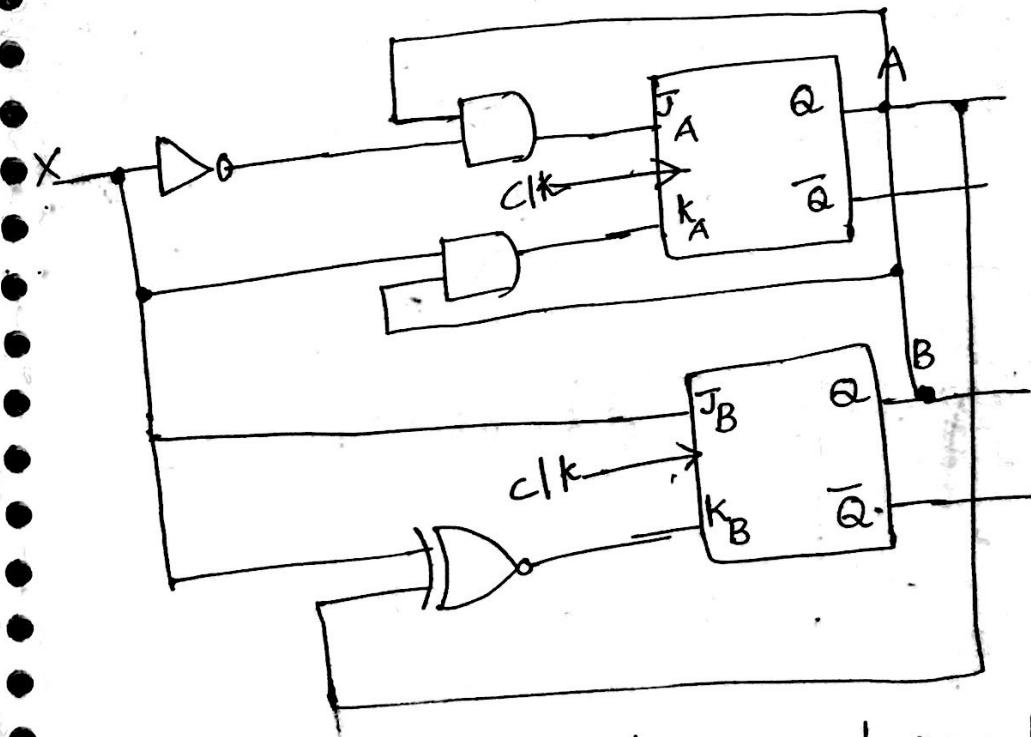
$$J_B = X$$

		J	B		
		00	01	11	10
A		0	X	X	(X)
1		X			

$$K_A = B \cdot X$$

		J	B		
		00	01	11	10
A		0	X	X	
1		X	(X)	X	

$$K_B = AX + \bar{A}\bar{X} = A \odot X$$



For the same state diagram design the sequential o/p (extra)
Circuit using D FF's :-

PS	I/P	NS	+DB	y
A 0	B 0	x 0	A 0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

$$\therefore Q_{n+1} = D$$

K-map :-

	Bx	00	01	11	10
A	0	1	1	1	0
	1	0	1	0	1

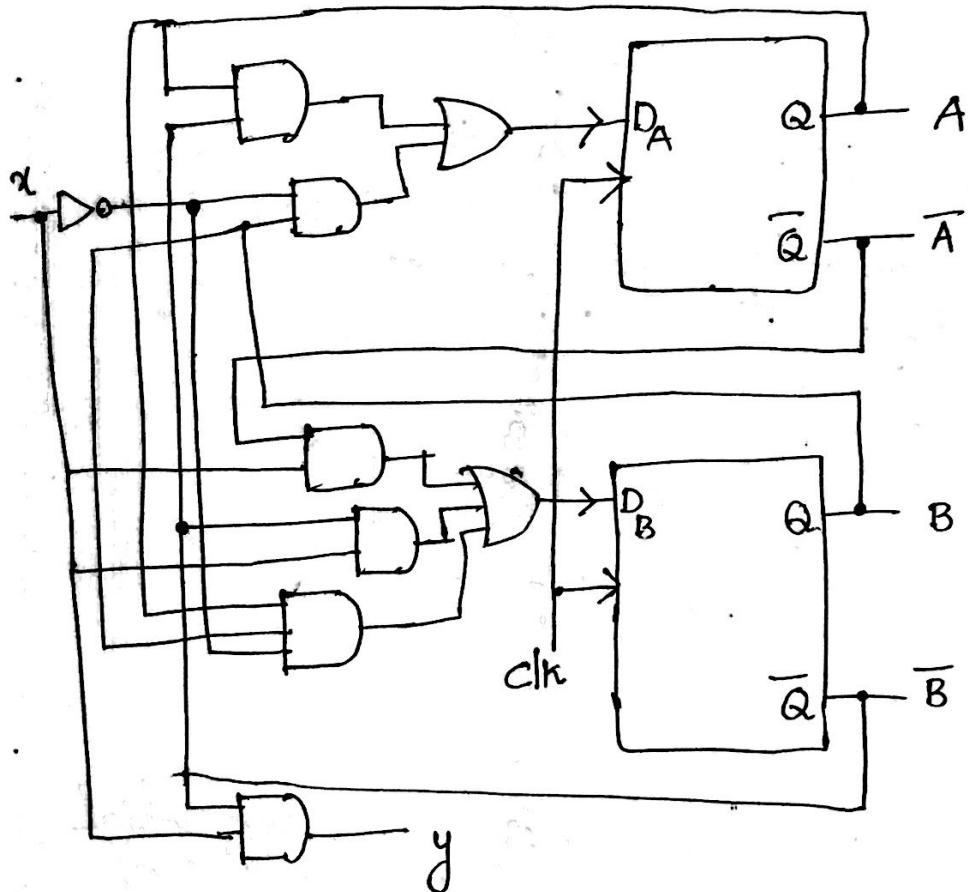
$$D_A = A^+ = AB' + B \cdot x'$$

	Bx	00	01	11	10
A	0	1	0	1	0
	1	0	1	0	1

	Bx	00	01	11	10
A	0	1	1	1	0
	1	0	1	0	1

$$D_B = B^+ = \overline{A} \cdot x + \overline{B} \cdot x \\ + A \cdot B \cdot \overline{x}$$

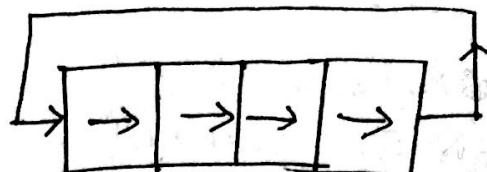
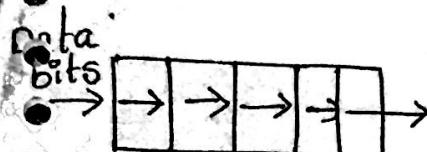
$$y = \overline{B} \cdot x$$



Registers :-

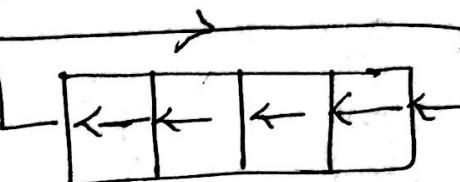
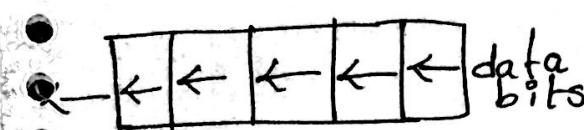
- The group of flipflops used to store the information is called registers.
- we can WRITE a data by using SI, PI
- we can READ a data by using SO, PO

Shift registers:- Here there is an movable data



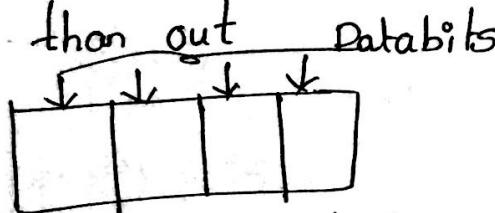
(b) Rotate right

(a) Serial shift right than out

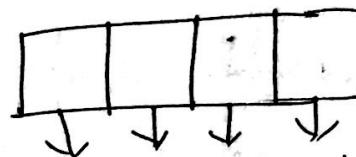


(d) Rotate Left

(c) Serial shift left than out



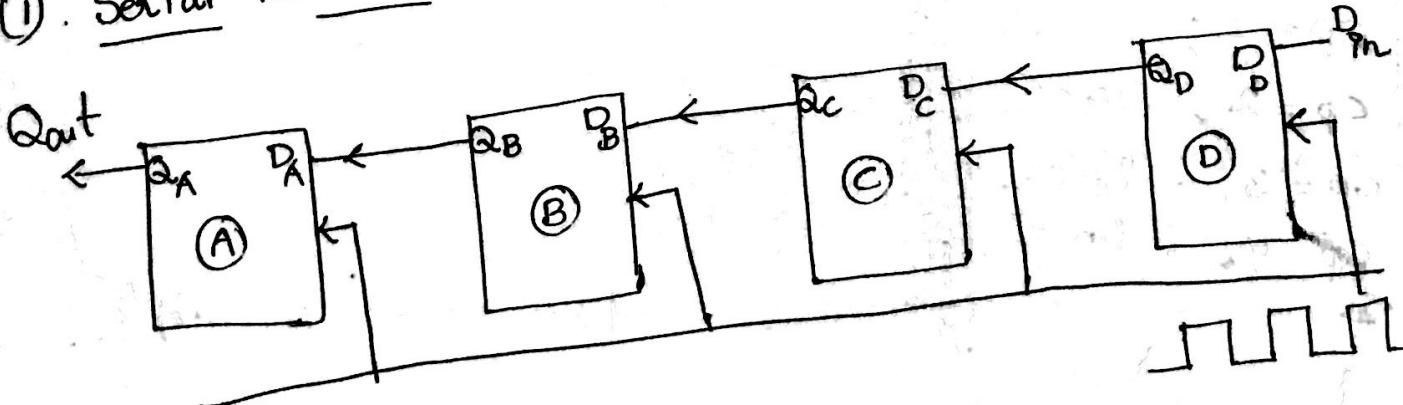
(e) parallel shift in



(f) parallel shift out

- A register which is going to shift right, left, sends and receives data serially and parallel is called universal shift registers.
- To design shift registers SR, JK, D flip-flops are used but not T-FF due to its toggling nature.

(1). Serial in Serial out (SISO) :-



$$\square = \rightarrow \quad \square \downarrow = \rightarrow^0 \quad \therefore Q_{n+1} = D$$

→ To send off 4 bit of information = 1111

1st clock pulse $D_{in} = 1$

$Cp = 1 \Rightarrow Q_A Q_B Q_C Q_D = 0001$

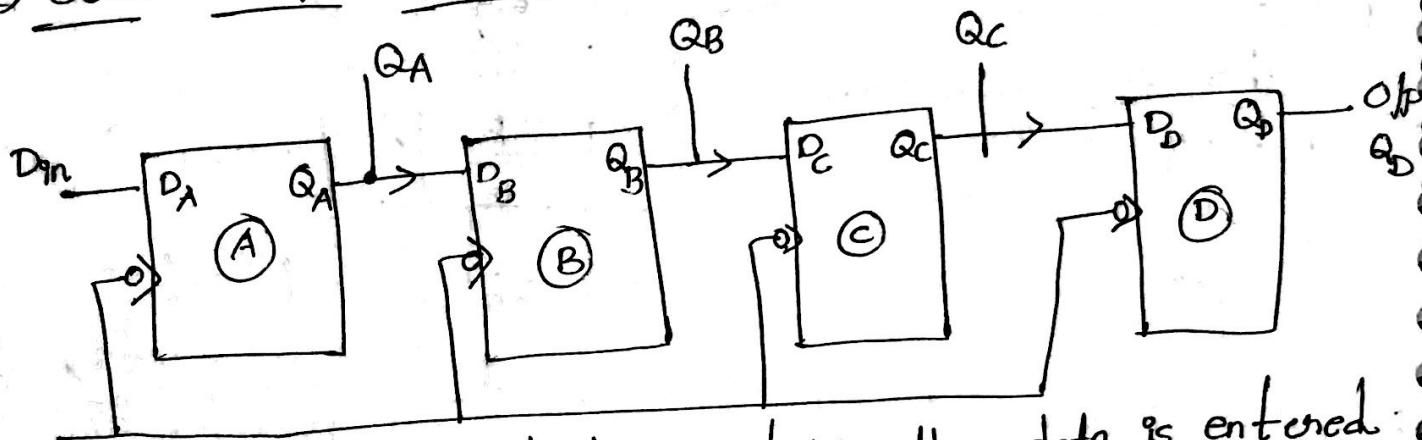
$Cp = 2 \Rightarrow Q_A Q_B Q_C Q_D = 0011$

$Cp = 3 \Rightarrow Q_A Q_B Q_C Q_D = 0111$

$Cp = 4 \Rightarrow Q_A Q_B Q_C Q_D = 1111$

→ Here Cp is serving (or) helpful to transfer the information.

(2) Serial in Parallel out :-



→ In this type of shift registers the data is entered same as the previous case and the collection of output is in parallel manner that means Q_A, Q_B, Q_C, Q_D are obtained simultaneously instead by bit-by-bit basis.

1 0 0 0 0 0 0 1

$Cp = 1, Q_A = 0$

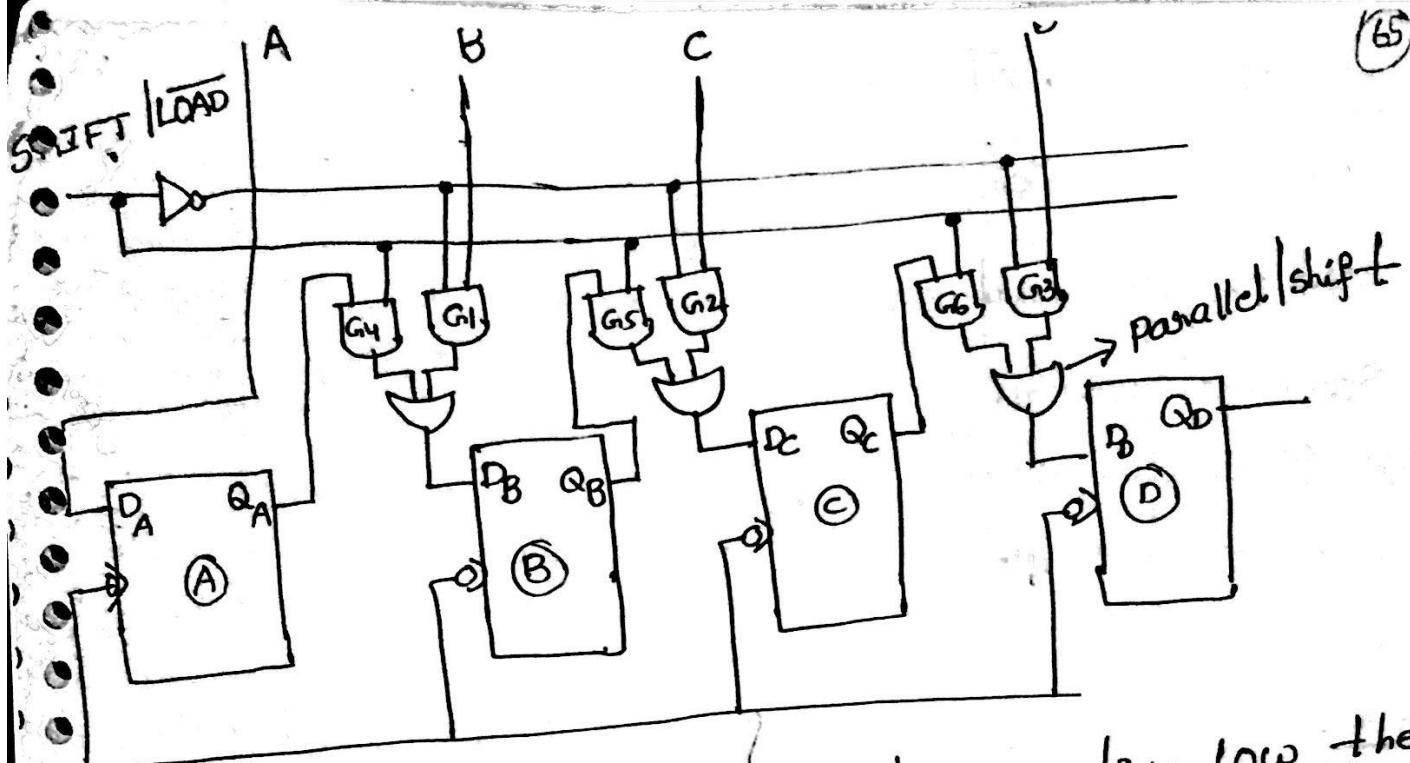
$Cp = 2, \text{ if } p \text{ at } D_B = 0100, Q_B = 0$

$Cp = 3, \text{ if } p \text{ at } D_C = 0010, Q_C = 0$

$Cp = 4, \text{ if } p \text{ at } D_D = 0001, Q_D = 1$

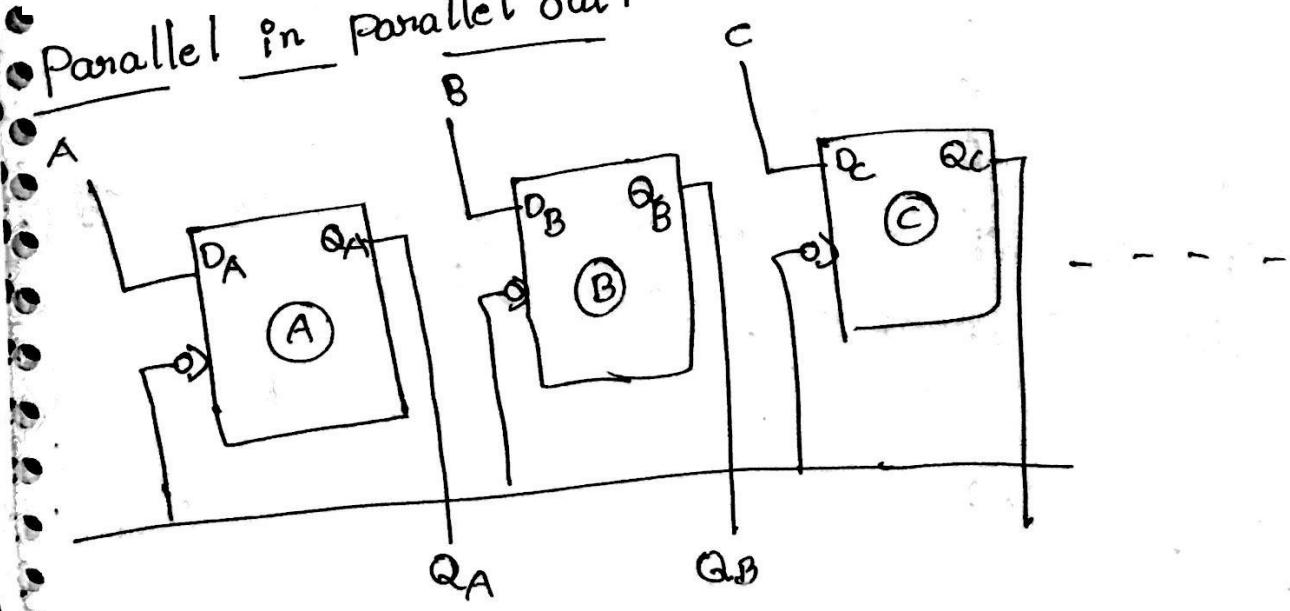
→ we are getting output parallelly as well as shifting is present.

3). Parallel in Serial out :-



- when the shift | Load signal is active low then the gates G1, G2, G3 are in enable mode and remaining AND gates G4, G5, G6 are in disabled mode & vice versa
- The function of the OR gate used in these registers is parallel entry data operation (or) shift operation. This can be done by the inputs obtained from above AND gates.

Parallel in parallel out :-



Applications of shift registers:-

(66)

1. Serial to Parallel Conversion (spatial to temporal code conversion)
2. parallel to serial conversion (temporal to spatial code conversion)
3. Sequence generator.
4. Multiplication and division.
5. Ring counter and Twisted ring counter.
6. Digital delay line (SIPO)

*** :-

1) Left shift operation is nothing but multiplied by 2.

$L \leftarrow R$

Eg: Q₃ Q₂ Q₁ Q₀

$$0 \quad 1 \quad 0 \quad 1 = 5$$

$$\begin{array}{cccc} & \swarrow & \downarrow & \searrow \\ 1 & 0 & 1 & 0 \end{array} = 10$$

→ Shift left by n-positions is equivalent to multiplication by 2^n .

$$\text{eg: } \underset{(n)}{2} = 0010$$

$$\begin{array}{c} \text{no. of positions} \\ 2 \times 2 = 2 \times 2 = 8 \\ \times 2^n \end{array}$$

$n = \text{given number}$

$$\begin{array}{cccc} 0 & 0 & 1 & 0 \\ \swarrow & \downarrow & \searrow & \downarrow \\ 0 & 1 & 0 & 0 \\ \swarrow & \downarrow & \searrow & \downarrow \\ 1 & 0 & 0 & 0 \end{array} = 8$$

1st
2nd

$$2 + 2 =$$

2). a) If LSB bit = 0, then right shift operation by one position is same as Division by 2.

Eg: Q₃ Q₂ Q₁ Q₀ (LSB)

$L \rightarrow R$

$$\begin{array}{cccc} 1 & 0 & 1 & 0 \\ \searrow & \downarrow & \searrow & \downarrow \\ 0 & 1 & 0 & 1 \end{array} = 5$$

(b) If $LSB = 1$, then right shift operation gives integer division by 2

Eg.: $Q_3 \ Q_2 \ Q_1 \ Q_0$

$$\begin{array}{cccc} 0 & 1 & 0 & 1 \\ \rightarrow & \rightarrow & & \\ 0 & 0 & 1 & 0 \end{array} = 5$$

$$= 2 \text{ (instead of } 2.5)$$

→ Delay line:

a) $SISO = (2n-1) T_c = 7T_c$

b) $SIP0 = n \cdot T_c = 8T_c$

c) $PISO = (n-1) T_c = 3T_c$

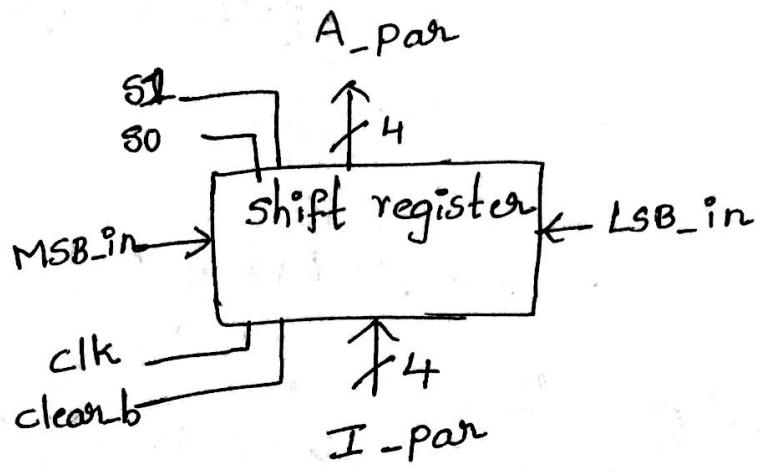
universal shift register:-

→ The most general shift register has the following capabilities:

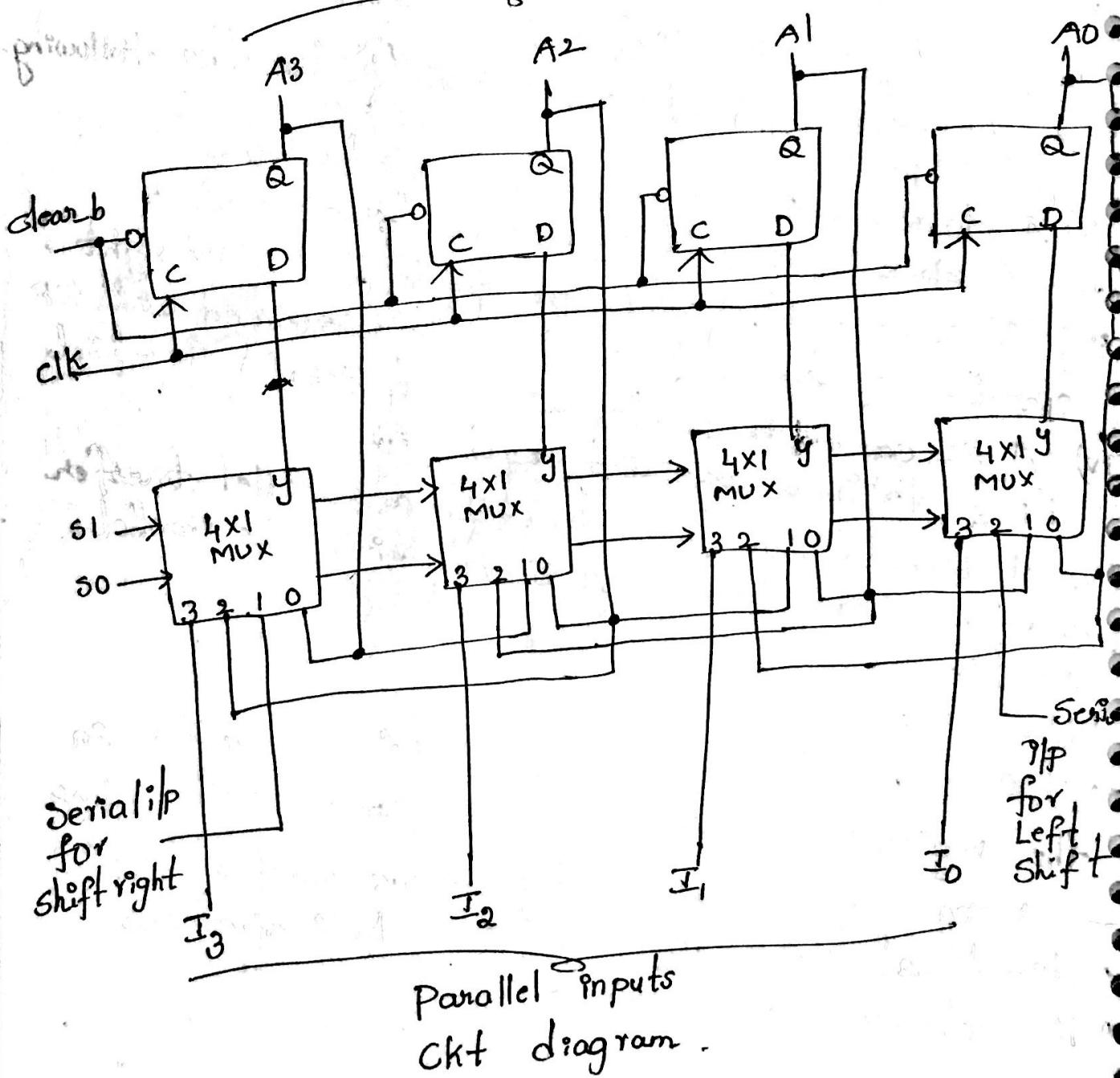
- i) A clear control to clear the register to 0.
 - ii) A clock input to synchronize the operations
 - iii) A shift right control to enable the shift right operation & serial IP & OP lines associated with SR.
 - iv) A SL control to enable shift left operation & serial IP & OP lines associated with SL.
 - v) A Parallel Load control to enable a parallel transfer and n input lines associated with the parallel transfer.
 - vi) n Parallel OP lines.
 - vii) A control state that leaves the information in the register unchanged in response to the clock.
- A reg capable of shifting in one direction → unidirectional shift register & in both directions = bidirectional shift register.

$I = b$ bits

(68)



Block diagram Parallel o/p's.



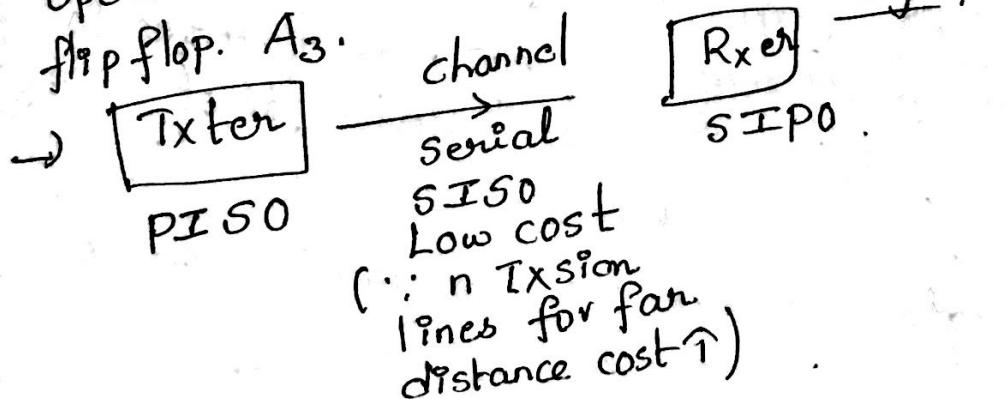
→ The ckt consists of 4 DFF's & 4 MUX's.
 The 4 MUX's have a common Selection lines S1 S0.
 I/P 0 in each mux is selected when $S1S0 = 00$,
 I/P 1 " " " " " " " " S1S0 = 01 &
 so on.
 So the 8-nits control the mode of operation.

Mode	control	Reg operation
S1	00	No change
0	01	shift right
0	10	shift left
1	11	Parallel Load

→. when $S150 = 00$, the present value of the reg is applied to the D1/p's of the FFS. This condition

forms a path from the output of each FF into the i/p of the same FF. (o/p recirculates to the i/p). The next clock edge transfers into each FF the old previously ie, no change.

- The next clock edge, the binary value it held previously ie, no change.
- $S_1 S_0 = 01$, terminal 1 of MUX 1/p's has a path to the D 1/p's of the FF's. This causes a shift right operation with the serial input transferred into



Counters :-
→ The counter is driven by a clock signal and can be used to count the number of clock cycles. [frequency divider ckt]

Ring counter :- Shift register can be used as ring counter when Q_0 o/p terminal is connected to serial i/p terminal.

→ An n-bit ring counter can have "n" different output states. It can count n-clock pulses.

Twisted ring counter :- It is also called Johnson ring counter. It is formed when \bar{Q}_0 o/p terminal is connected to the serial input terminal of the shift register. It can have max of 2^n different o/p states.

→ The largest binary number that can be represented by an n-bit counter has a decimal equivalent of $2^n - 1$ Eg :- $n=3, 2^3 - 1 = 7$

→ A counter having n FF's can have 2^n o/p states
→ It can count either in the up mode / down mode

→ The Modulus of a counter is the total no. of states Eg :- Mod-8 (000 to 111)

→ The o/p signal frequency of mod-n counter is $\frac{1}{n}$ th of the i/p clk frequency. Hence that counter is also called $\div n$ counter

→ The no. of FF's required to construct Mod-10 (or) $\div 10$ counter = 4. (decade counter)

→ formula : $2^{n-1} < N \leq 2^n$ $N = \text{Mod Number}$
 $n = \text{FF's}$

→ Synchronous $\begin{cases} \text{Series carry} \\ \text{Parallel carry} \end{cases}$

→ Based on the nature of the clock pulse applied
The counters are classified into :

(7)) Asynchronous (Ripple (carry)) | Series Counter.

→ In asynchronous counter the output of first FF is given as input/
(clock) to the next FF as a clock pulse. This processes continues such counters are known as asynchronous counters.

2) Synchronous counter:- In synchronous counter clock pulse is applied simultaneously to all the flipflops.

Differences between Asynch and synchronous :-

Asynchronous

1. In this type of counters FF's are connected in such a way that the output of first FF derives the clock for the next FF.
2. All the FF's are not clocked simultaneously.
3. Logic ckt is very simple even for more no. of states (2^n).
4. Main drawback of this counter is there low speed as the clock is propagated through no. of FF's before it reaches last FF

Synchronous

1. In this type there is no connection between the output of first FF and clock ip of next FF.
2. All the FF's are clocked simultaneously.
3. Design involves complex logic ckt as the no. of states increases.
4. They are performed with high speed even when the no. of FF's are increased.

1) Design 4-bit asynchronous counter using JKFF's? (72)

Sol:- 4-bit = 2^4 states = 16 states
= 2^n = 4 FF's are required.

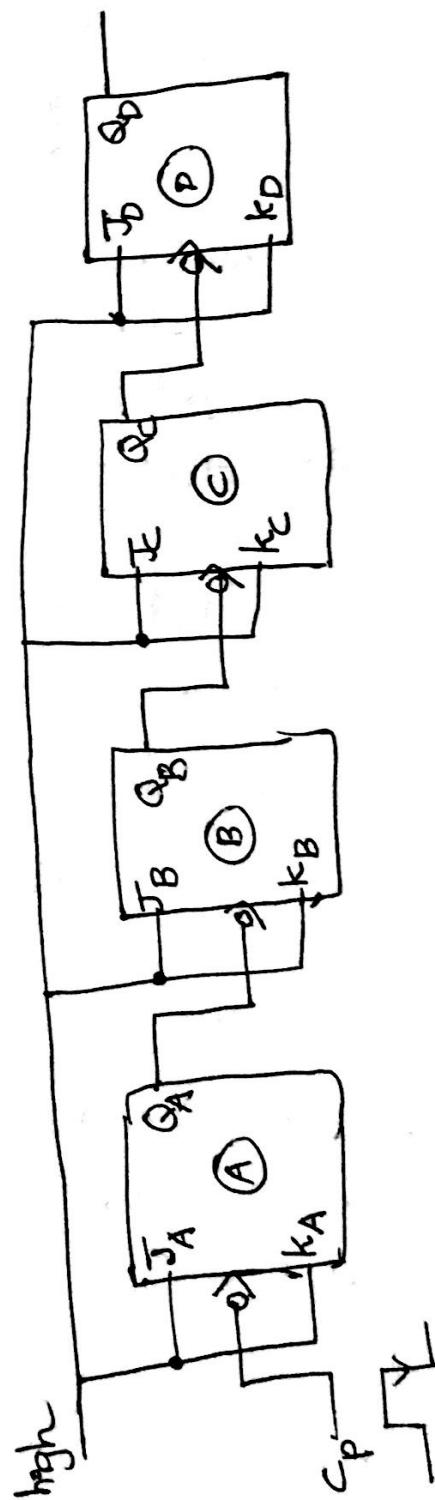
Excitation table for JK FF

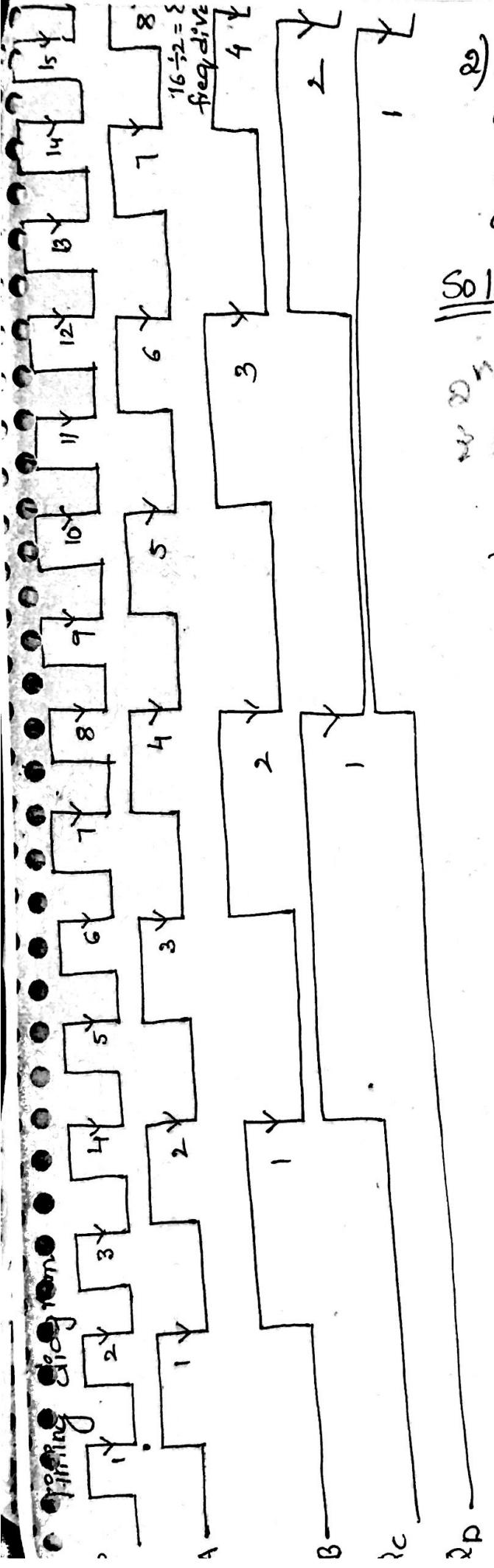
Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

State Table :-

CP	P/S			
	Q_D	Q_C	Q_B	Q_A
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Here NS of 1st FF is given as Clk for 2nd FF so, there is no need of excitation table





2). Design a MOD-5 synchronous counter using JK flip-flops and implement it. Also - construct a timing diagram.

Sol:- MOD - N

$$2^n \geq N$$

$$2^1 \geq N \quad 2^1 \geq 5 \quad \text{Not possible}$$

$$2^2 \geq N \quad 2^2 \geq 5 \quad \text{possible} \checkmark$$

So, we require 3 Flip Flops to design MOD-5 synchronous counter.

Excitation table for JK :-

Q _n	Q _{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

P _S	N _S
0.	1
1	2
2	3
3	4
4	0
5	X
6	X
7	X

State Table:-

PS			NS			Flip-Flop Inputs					
Q_C	Q_B	Q_A	Q_{C+1}	Q_{B+1}	Q_{A+1}	J_C	k_C	J_B	k_B	J_A	k_A
0	0	0	0	0	1	0	x	0	x	1	x
0	0	1	0	0	0	0	x	1	x	x	1
0	1	0	0	1	1	0	x	x	0	1	x
0	1	1	1	0	0	1	x	x	1	x	1
1	0	0	0	0	0	x	1	0	x	0	x
1	0	1	x	x	x	x	x	x	xx	x	x
1	1	0	x	x	x	x	x	x	xx	x	x
1	1	1	x	x	x	x	x	x	xx	x	x

1st

2nd level

3rd level

K-map :-

		$Q_B Q_A$
		00 01 11 10
Q_C	0	0 0 1 0
	1	x x x x

		$Q_B Q_A$
		00 01 11 10
Q_C	0	x x x x
	1	1 x x x

$$J_C = Q_B Q_A$$

		$Q_B Q_A$
		00 01 11 10
Q_C	0	0 1 x x
	1	0 x x x

$$J_B = Q_A$$

		$Q_B Q_A$
		00 01 11 10
Q_C	0	1 x x x
	1	0 x x x

$$J_A = \overline{Q_C}$$

$$k_C = 1$$

		$Q_B Q_A$
		00 01 11 10
Q_C	0	x x 1 0
	1	x x x x

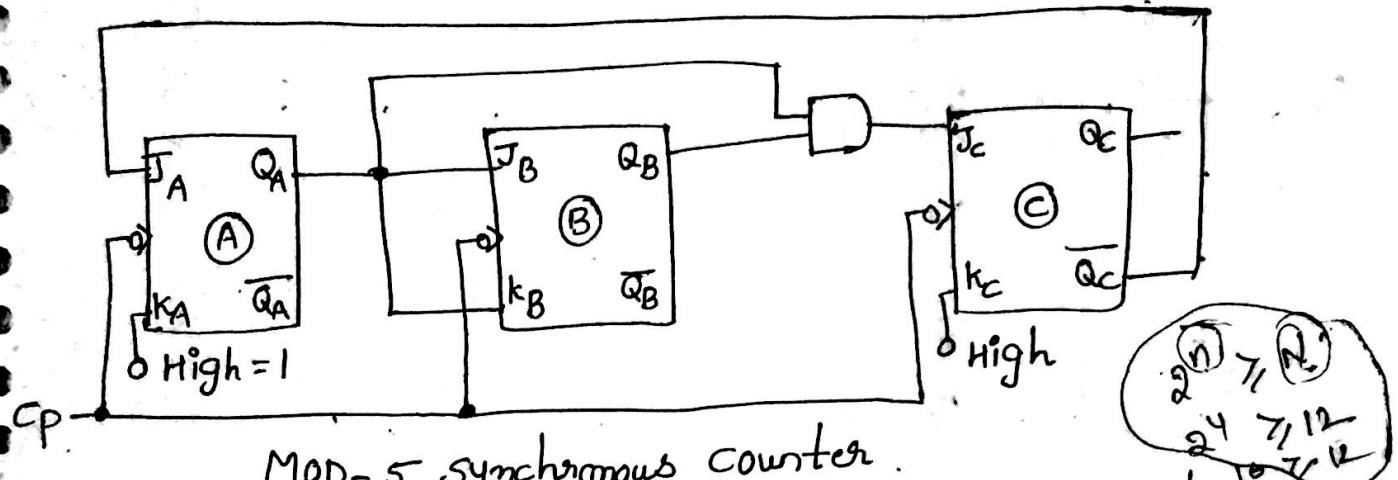
$$k_B = Q_A$$

		$Q_B Q_A$
		00 01 11 10
Q_C	0	x 1 1 x
	1	x x x x

$$k_A = 1$$

Logic diagram:-

(75)



MOD-5 synchronous counter.

Synchronous decade / MOD-10 counter :- $2^n \geq N$, $2^4 \geq 10$
 $FF's = 4$.

State Table :-

PS	NS				FF inputs							
	Q_D	Q_C	Q_B	Q_A	Q_{D+1}	Q_{C+1}	Q_{B+1}	Q_{A+1}	T_D	T_C	T_B	T_A
0 0 0 0	0	0	0	0	0	0	0	1	0	0	0	1
0 0 0 1	0	0	0	1	0	0	1	0	0	0	1	1
0 0 1 0	0	0	0	1	0	0	1	1	0	0	0	1
0 0 1 1	0	0	1	1	0	1	0	0	0	1	1	1
0 0 0 0	0	0	0	0	0	1	0	0	0	0	0	1
0 0 0 1	0	0	0	1	0	1	1	0	0	0	0	1
0 0 1 0	0	0	1	0	0	1	0	1	0	0	1	1
0 0 1 1	0	0	1	1	0	1	1	1	0	0	0	1
0 1 0 0	0	1	0	0	1	0	0	0	0	0	1	1
0 1 0 1	0	1	0	1	1	0	1	0	0	0	0	1
0 1 1 0	0	1	1	0	1	1	1	1	1	1	1	1
0 1 1 1	1	0	0	0	0	0	0	0	0	0	0	1
1 0 0 0	1	0	0	0	0	0	0	1	0	0	0	1
1 0 0 1	0	1	0	0	0	0	0	0	1	0	0	1
1 0 1 0	x	x	x	x	x	x	x	x	x	x	x	x
1 0 1 1	x	x	x	x	x	x	x	x	x	x	x	x
1 1 0 0	x	x	x	x	x	x	x	x	x	x	x	x
1 1 0 1	x	x	x	x	x	x	x	x	x	x	x	x
1 1 1 0	x	x	x	x	x	x	x	x	x	x	x	x
1 1 1 1	x	x	x	x	x	x	x	x	x	x	x	x

-map:-

$$T_D = Q_A Q_D + Q_A Q_B Q_C$$

		$Q_B Q_A$	00	01	11	10
		$Q_D Q_C$	00	01	11	10
00	00	0	0	0	0	0
01	01	0	0	1	0	0
11	11	X	X	X	X	X
10	10	0	1	X	X	X

$$T_C = Q_A Q_B$$

		$Q_B Q_A$	00	01	11	10
		$Q_D Q_C$	00	01	11	10
00	00	0	0	1	0	0
01	01	0	0	1	0	0
11	11	X	X	X	X	X
10	10	0	0	X	X	X

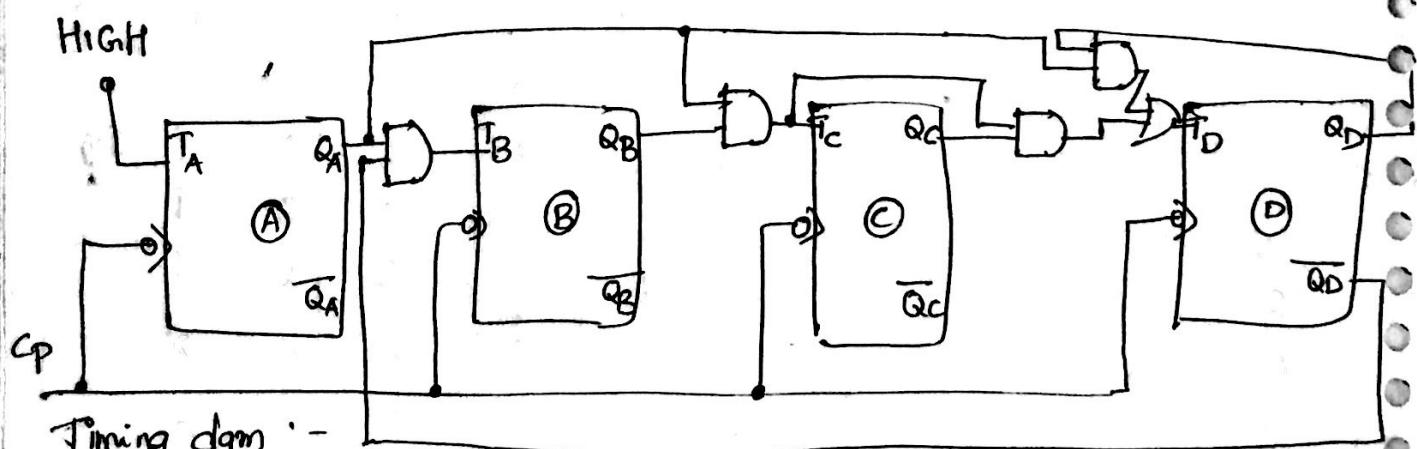
		$Q_B Q_A$	00	01	11	10
		$Q_D Q_C$	00	01	11	10
00	00	0	1	1	0	0
01	01	0	1	1	0	0
11	11	X	X	X	X	X
10	10	0	0	X	X	X

		$Q_B Q_A$	00	01	11	10
		$Q_D Q_C$	00	01	11	10
00	00	1	1	1	1	1
01	01	1	1	1	1	1
11	11	X	X	X	X	X
10	10	1	1	1	1	1

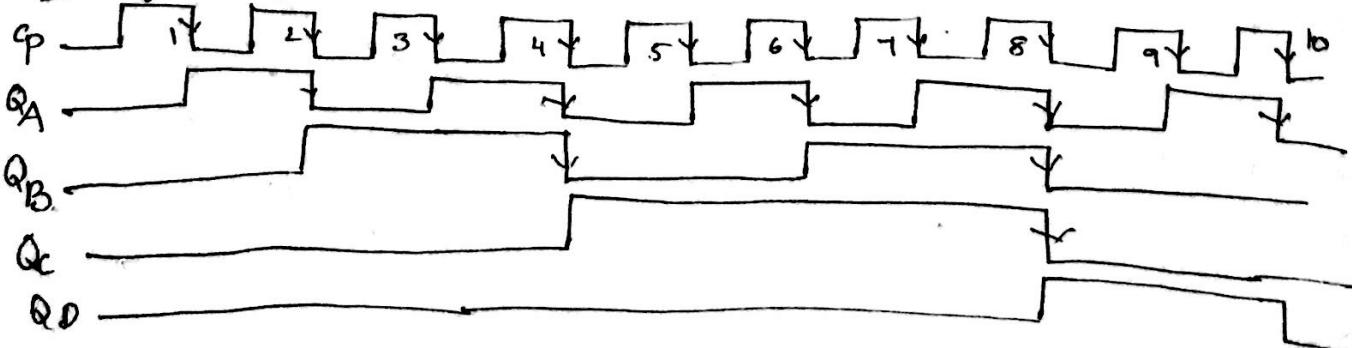
$$T_B = Q_A \bar{Q}_D$$

$$T_A = 1$$

Logic diagram :-



Timing diagm :-



4) Design of UP/DOWN synchronous counters:-

(78)

- An up/down counter is also called as bidirectional counter. Usually up/down operation of the counter is controlled by UP/DOWN signal.
- When this signal is high, counter goes through up sequence i.e., 0, 1, 2, ... n. When UP/DOWN = 0 counter follows reverse sequence i.e., n, n-1, n-2, ..., 1, 0.
- For 3-bit counters these sequences are:
0, 1, 2, 3, 4, 5, 6, 7 for up operation & 7, 6, 5, 4, 3, 2, 1, 0 for DOWN operation.

State Table :-

C_P	UP	Q_C	Q_B	Q_A	DOWN
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	0	0
7	1	1	1	0	1

K-map :-

$UD\ Q_C$	$QB\ QA$	00	01	11	10
00	1	0	0	0	0
01	1	0	0	0	0
11	0	1	1	0	0
10	0	1	1	0	1

$$T_C = \overline{UD} \overline{QB} \overline{QA} + UD QB QA$$

$UD\ Q_A$	$QB\ QA$	00	01	11	10
00	1	0	0	1	
01	1	0	0	1	
11	0	1	1	0	
10	0	1	1	0	

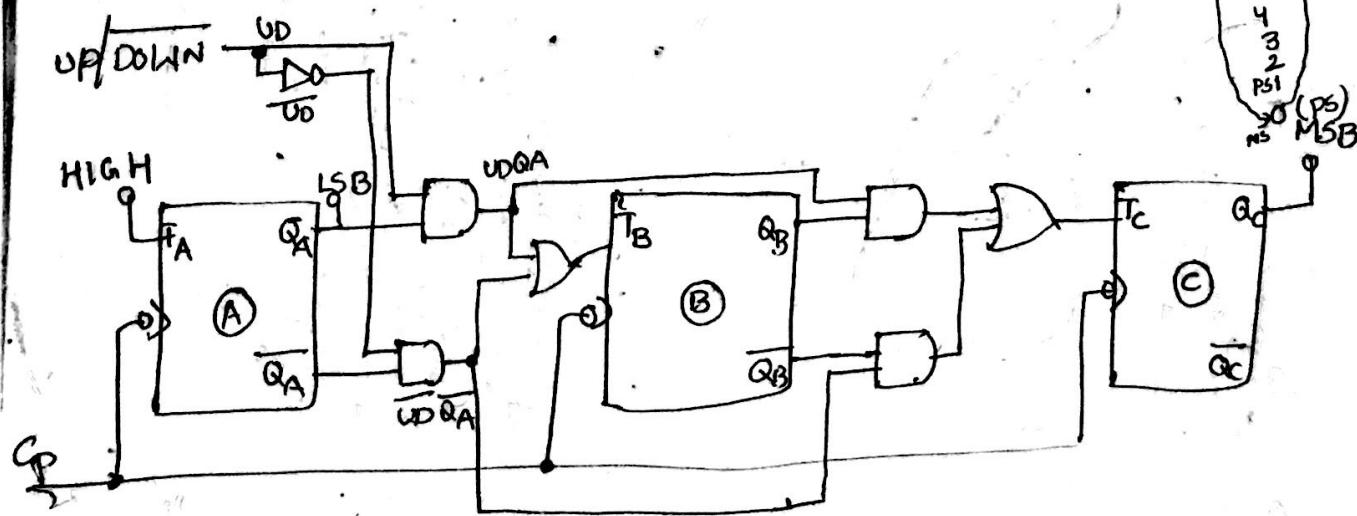
$$T_B = \overline{UD} \overline{QA} + UD QA$$

$UD\ Q_C$	$QB\ QA$	00	01	11	10
00	1	1	1	1	
01	1	1	1	1	
11	1	1	1	1	
10	1	1	1	1	

$$T_A = 1$$

Tr :-

I/P UP/DOWN (UD)	PS			NS			FFi/p's		
	Q_C	Q_B	Q_A	Q_{C+1}	Q_{B+1}	Q_{A+1}	T_C	T_B	T_A
0	0	0	0	1	1	1	1	1	1
0	0	0	1	0	0	0	0	0	1
0	0	1	0	0	0	1	0	1	1
0	0	1	1	0	1	0	0	0	1
0	1	0	0	0	1	1	1	1	1
0	1	0	1	0	0	0	0	0	1
0	1	1	0	1	0	1	0	1	1
0	1	1	1	1	1	0	0	0	1
1	0	0	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0	0	1
1	0	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	0	1
1	1	0	1	1	0	1	0	0	1
1	1	1	0	1	1	1	0	0	1
1	1	1	1	0	0	0	1	1	1



(80)

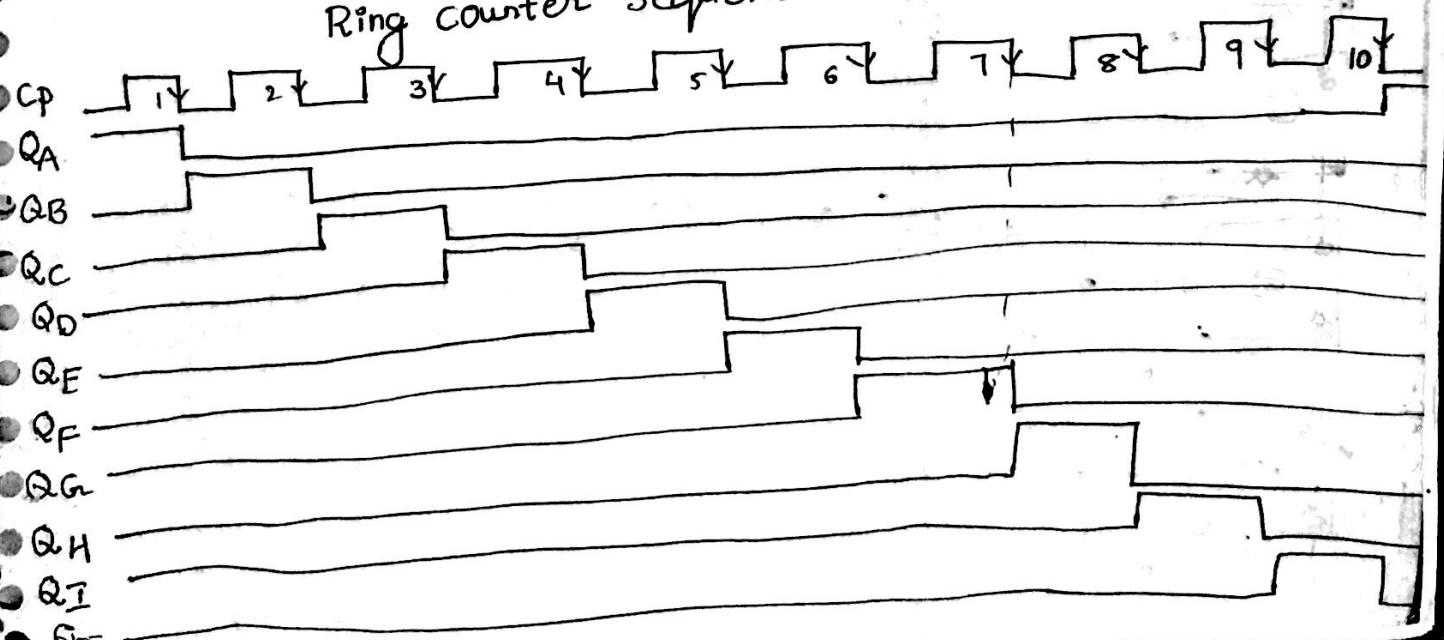
Ring counter :- → The Q o/p of each stage is connected to the D input of the next stage and the output of last stage is fed back to the input of first stage.

→ The CLR followed by PRE makes the o/p of first stage to '1'. & remaining outputs are zero. $Q_A = 1, Q_B = \dots Q_J = 0$.

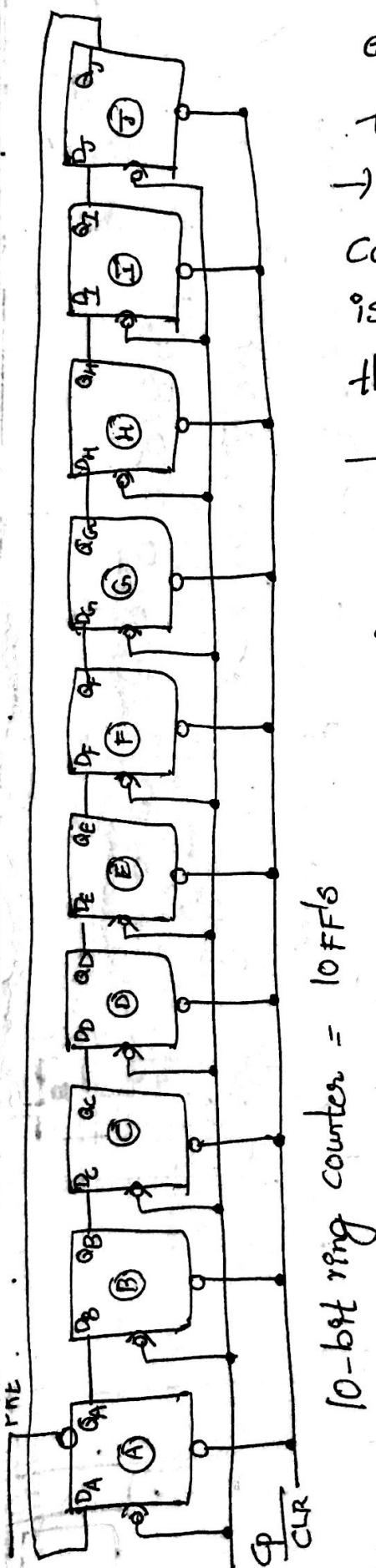
→ The 1st clk pulse produces $Q_B = 1$ & remaining o/p's are zero. [10 bits] (0---9)

clk pulse	Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_I	Q_J
0	1	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0	1	0	0
7	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0

Ring counter sequence (10-bits).



Shift/ Johnson/Twisted Ring/Moebius Counter

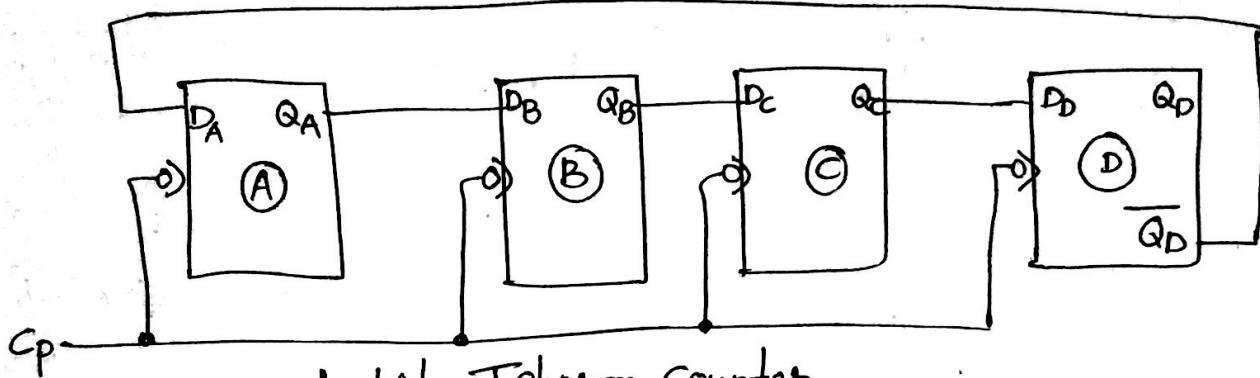
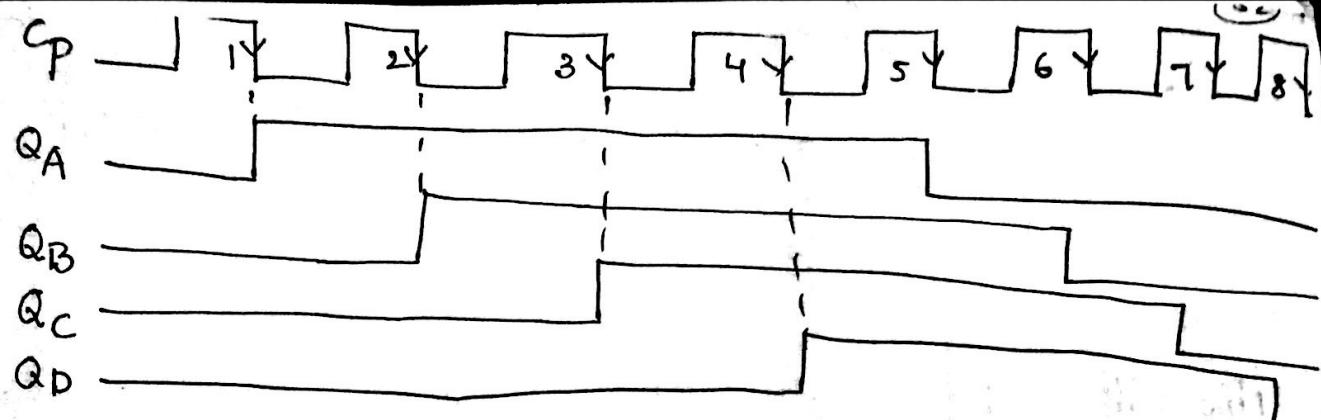


- In a Johnson counter, the Q o/p of each stage of flip-flop is connected to the D input of the next stage.
- The single exception is that the complement output of the last flip-flop is connected back to the D-input of the first FF.
- Initially the register (all FF's) is cleared. so all the outputs, $Q_A Q_B Q_C Q_D = 0000$. The o/p of last stage $Q_D = 0$. \therefore , complement output of last stage, $\overline{Q_D}$ is one
- This is connected back to the D input of first stage. so, $D_A = 1$.
- The 1st falling clock edge produces $Q_A = 1$ & $Q_B = 0$, $Q_C = 0$, $Q_D = 0$.
- The next clock pulse produces

$$Q_A = 1, Q_B = 1, Q_C = 0, Q_D = 0$$

C_p	Q_A	Q_B	Q_C	Q_D	$Q_D = 1$
0	0	0	0	0	0
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	1
5	0	1	1	1	1
6	0	0	1	1	1
7	0	0	0	1	1

4-bit
Johnson
Sequence



4-bit Johnson Counter

Hint :- n stage Johnson counter will produce 2^n
 n = no. of stages (FF's)

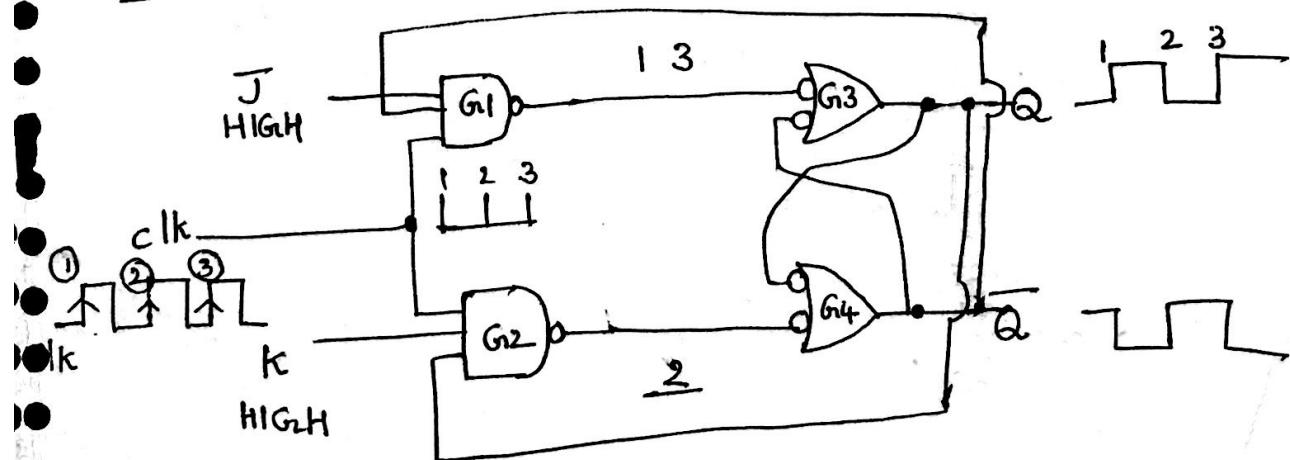
4 bit = 4 FF's = $2^n = 2 \times 4 = 8$ stages 0 to 7

5-bit = 5 FF's = $2^n = 2 \times 5 = 10$ stages 0 to 9

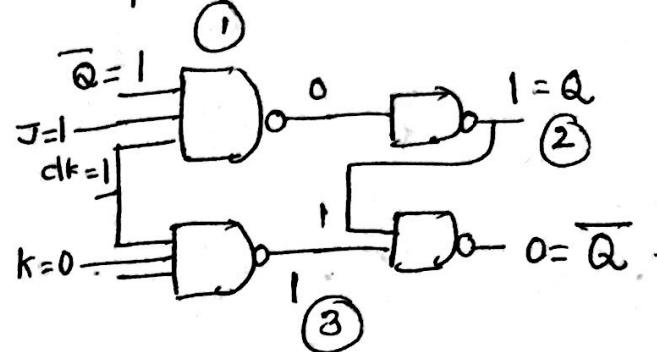
10-bit = 7 FF's = $2^n = 2 \times 7 = 14$ stages 0 to 19.

JK F/F :-

Rayleigh
Riccan

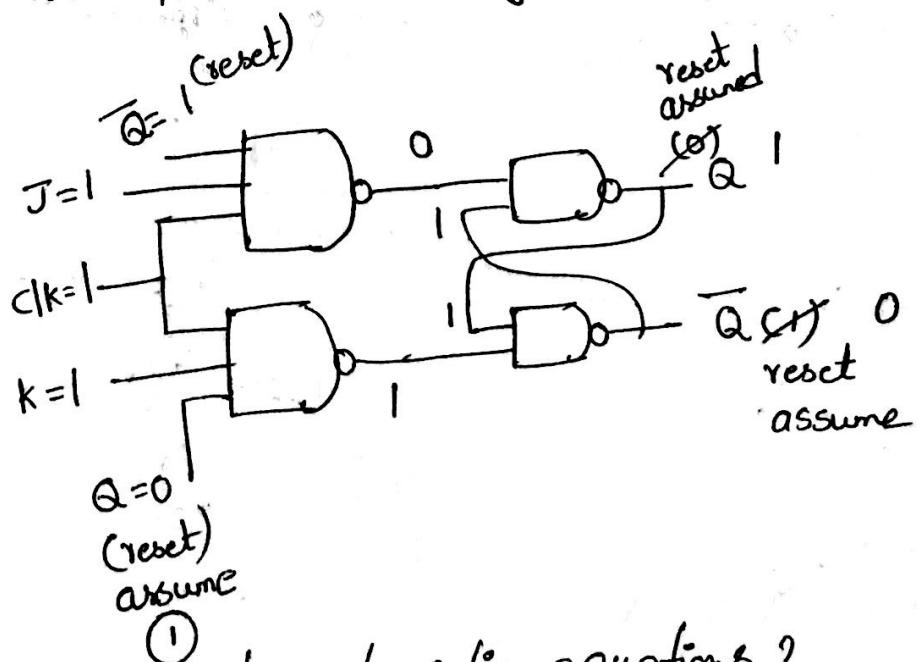


→ Let us assume that the FF is RESET and $J=1$, $K=0$. When a clk pulse occurs, a leading-edge spike indicated by ① is passed via gate G₁ because $\bar{Q}=1$, $J=1$. This will cause the latch portion of the FF to change to the SET state.



→ The FF is now SET. $J=0$, $K=1$, the next clock spike indicated by ② will pass through gate G₂ because $Q=1$ & $K=1$. \Rightarrow RESET.

→ The difference in operation occurs when both the J and K inputs are HIGH. To see this, assume that the FF is RESET. [$Q=0$, $\bar{Q}=1$]. The high on the \bar{Q} enables gate G₁, so the clock spike indicated by ③ passes through to set the FF.



→ How to obtain characteristic equations?
SOL: characteristic eqn's are obtained from characteristic

11102