SUPER CONDUCTIVITY

INTRODUCTION

Resistance or *resistivity* is the inherent property exhibited by materials. This is mainly due to the scattering of electrons while interacting with the positive ions (atoms) present in the materials. When temperature of material is decreased to a low value, then due to lower energy, scattering of electrons decreases and as a result resistance or resistivity decreases. Then the conductivity increases.

The phenomenon of attaining zero resistivity or infinite conductivity at low temperature is known as super conductivity. The material becomes a super conductor and it will be in super-conducting state.

Super conductivity was first observed by Kammerlingh Onnes in the case of mercury. When temperature of mercury is decreased then the resistance also decreases and it is nearly zero at 4.2 K temperature as shown in Fig. 9.1.

The temperature at which the material undergoes a transistion from normal state to super-conducting state is known as *critical temperature* or *transition temperature* (T_c). Different materials will have different T_c values.

Example	Aluminium	$T_c = 1.19\,\text{K}$
	Lead	$T_c = 7.2\,\text{K}$
	Tungsten	$T_c = 0.01\,\text{K}$
	Tin	$T_c = 0.39\,\text{K}$
	Cadmium	$T_c = 0.55\,\text{K}$

9.1 GENERAL PROPERTIES

1. Super conductivity is a low-temperature phenomenon.
2. The transition from normal state to super-conducting state occurs below the critical temperature.

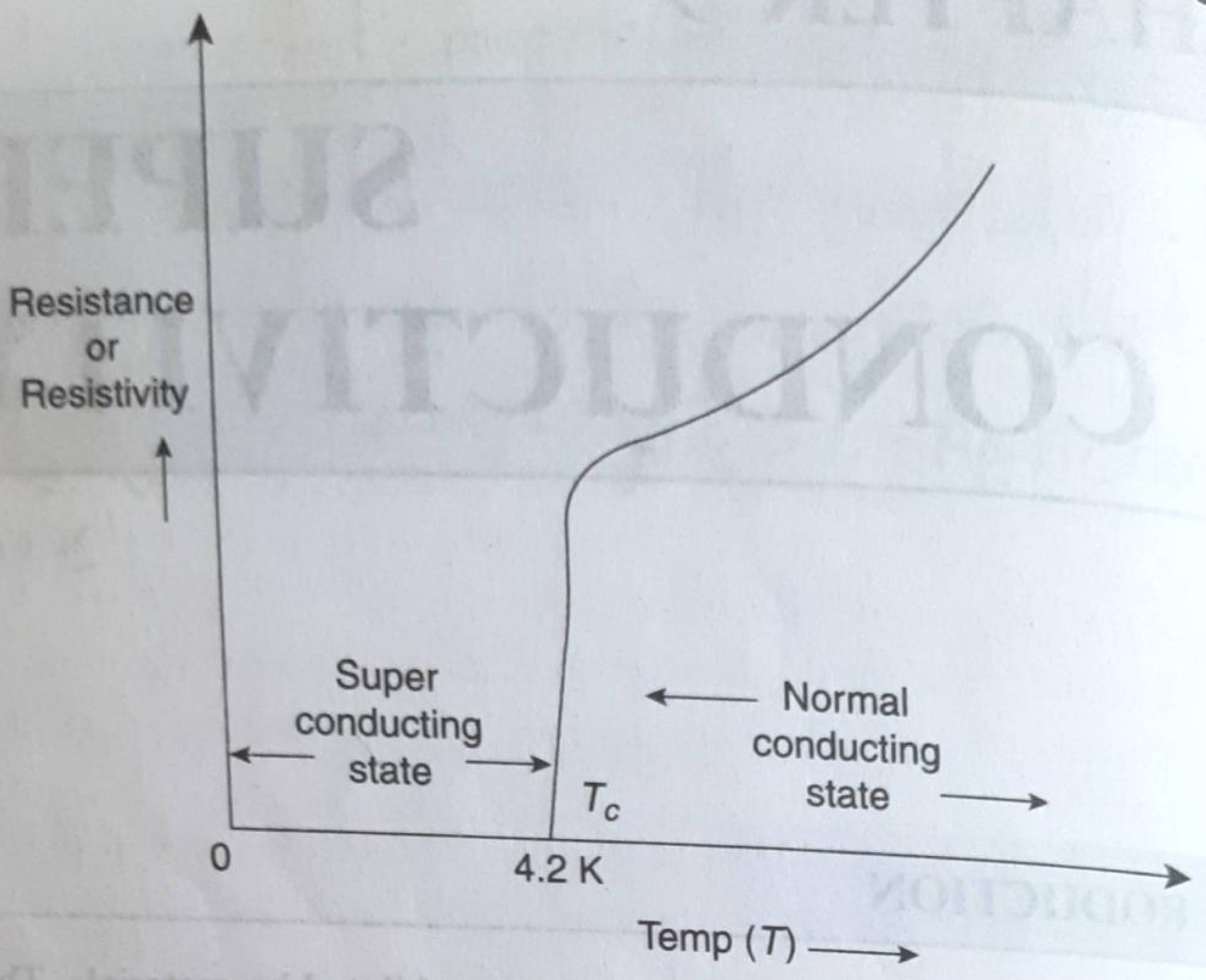


Figure 9.1 Resistivity of mercury

3. Different materials will have different critical temperatures.
4. The current once set up in a super conductor persists for a long time due to zero resistivity.
5. Super conductors do not allow magnetic field (magnetic lines) through them and behave as a diamagnetic. This property of expulsion of magnetic field is known as Meissner effect.
6. The magnetic field at which a super conductor loses it's super conductivity and becomes a normal conductor is known as *critical magnetic field* H_c.
7. The induced current in a super conductor induces a magnetic field in it. If the magnetic field is equal to the critical magnetic field then it converts into a normal conductor. The current in it is known as critical current (I_c). If 'r' is the radius of the super conductor then

   $$I_c = 2\pi r H_c$$

   The current density at which it occurs is known as critical current density and is given by $J_c = I_c/A$, where A is the area of cross section of the super conductor.
8. Super conductivity occurs in metallic elements in which the number of valence electrons lies between 2 and 8.
9. Materials having high normal resistivities exhibit super conductivity.
10. Super-conducting materials are not good conductors at room temperature.

9.1.1 Critical Magnetic Field (H_c)

When a magnetic field is applied to a super conductor then for a particular value of applied field it loses super conductivity and becomes a normal conductor. The magnetic field for which a super conductor becomes a normal conductor is known as critical magnetic field (H_c) and is given by

$$H_c = H_0\left|1 - \left(\frac{T}{T_c}\right)^2\right|$$

where H_0 is the field required to destroy the super-conducting property at 0 K, T_c is the critical temperature of the super conductor and T is the temperature of the super conductor.

When $T = T_c$, then $H_c = 0$.

When $T = 0$ K then $H_c = H_0$.

The variations of H_c w.r.t. T is as shown in Fig. 9.2.

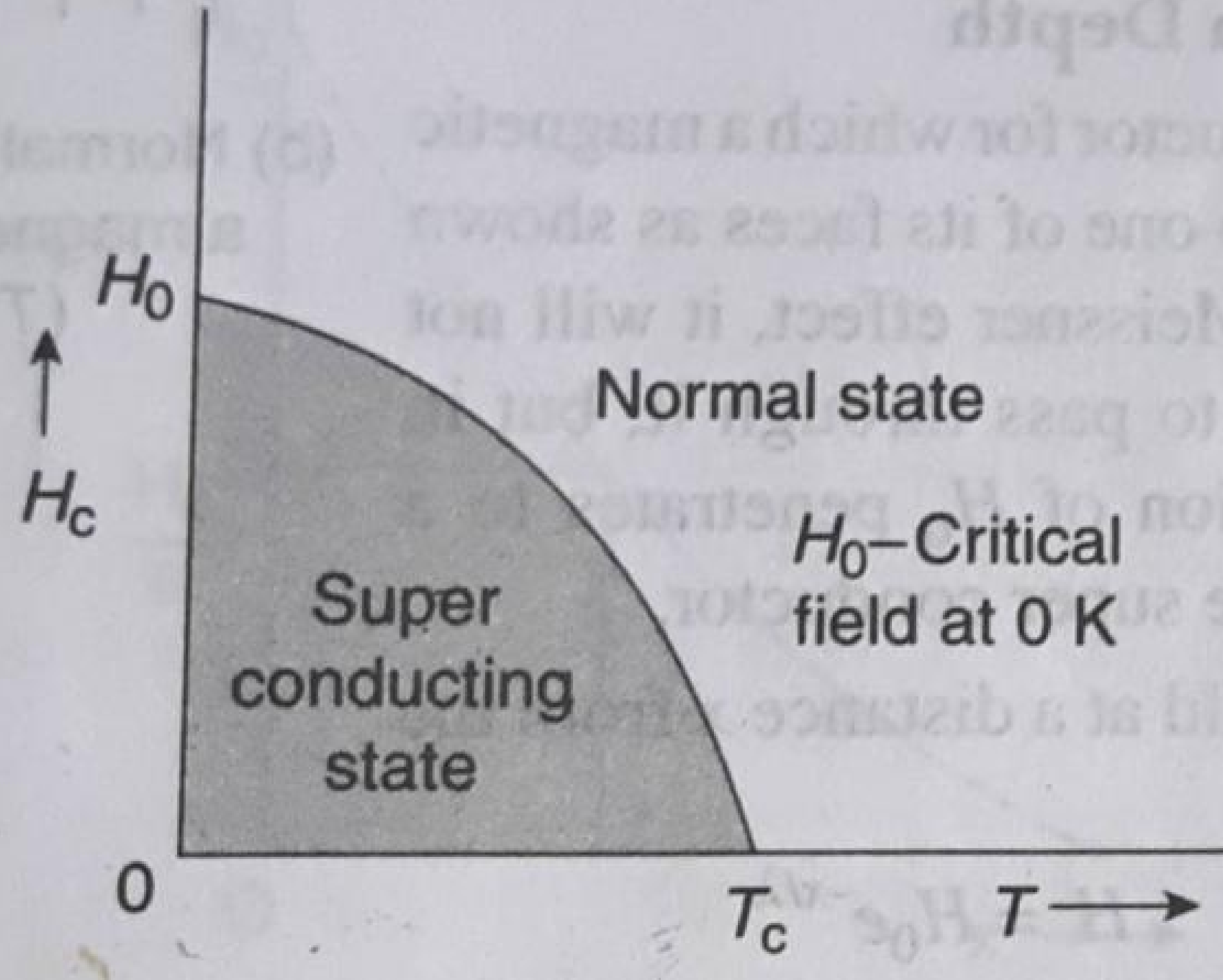


Figure 9.2 Variations of H_c with T

9.2 MEISSNER EFFECT

Consider a normal conductor at room temperature (Fig. 9.3a). When a magnetic field H is applied to it then it allows the magnetic lines to pass through it. Thus we have a magnetic induction field 'B' in a conductor (Fig. 9.3b). When the entire system is cooled so that $T < T_c$ then the normal conductor becomes a super conductor and it will not allow the magnetic lines to pass through it. It expels the magnetic lines. This effect, observed by Meissner, is known as Meissner effect. Thus, the super conductor does not allow the magnetic lines through it or expels the magnetic lines.

For a normal conductor, magnetic inductions field 'B' is given by

$$B = \mu_0(H + M)$$

where μ_0 is the permeability of free space or air,

M is magnetisation of the normal conductor

For a super conductor, $B = 0$

$$\mu_0(H + M) = 0$$

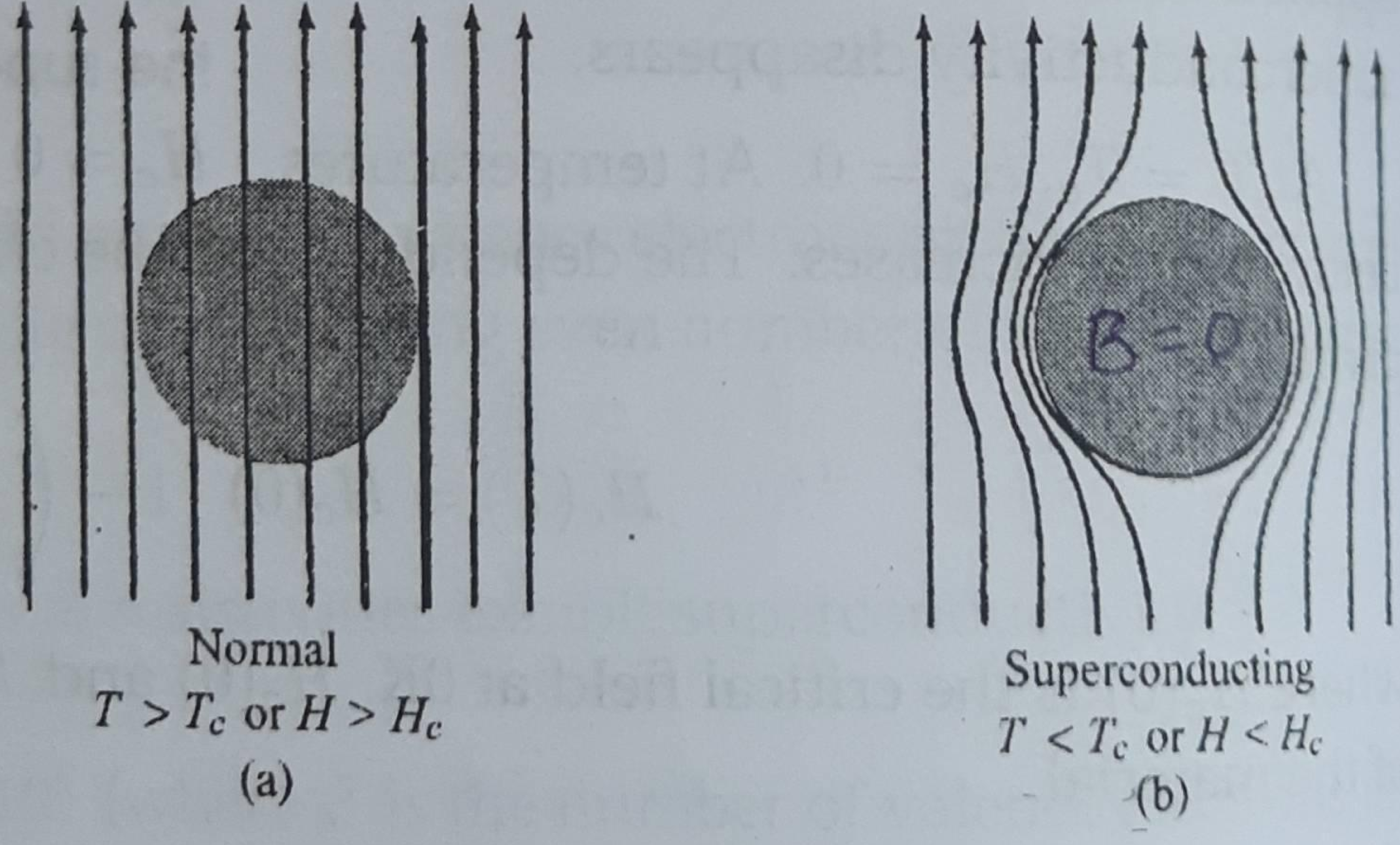
$$\boxed{H = -M}$$

i.e., applied magnetic field induces magnetisation in opposite direction.

Magnetic susceptibility $\chi = \frac{M}{H} = -1$

$$\boxed{\chi = -1}$$

The above results clearly indicate that a super conductor behaves as a perfect diamagnetic in the presence of a magnetic field. Meissner effect proved the above fact.



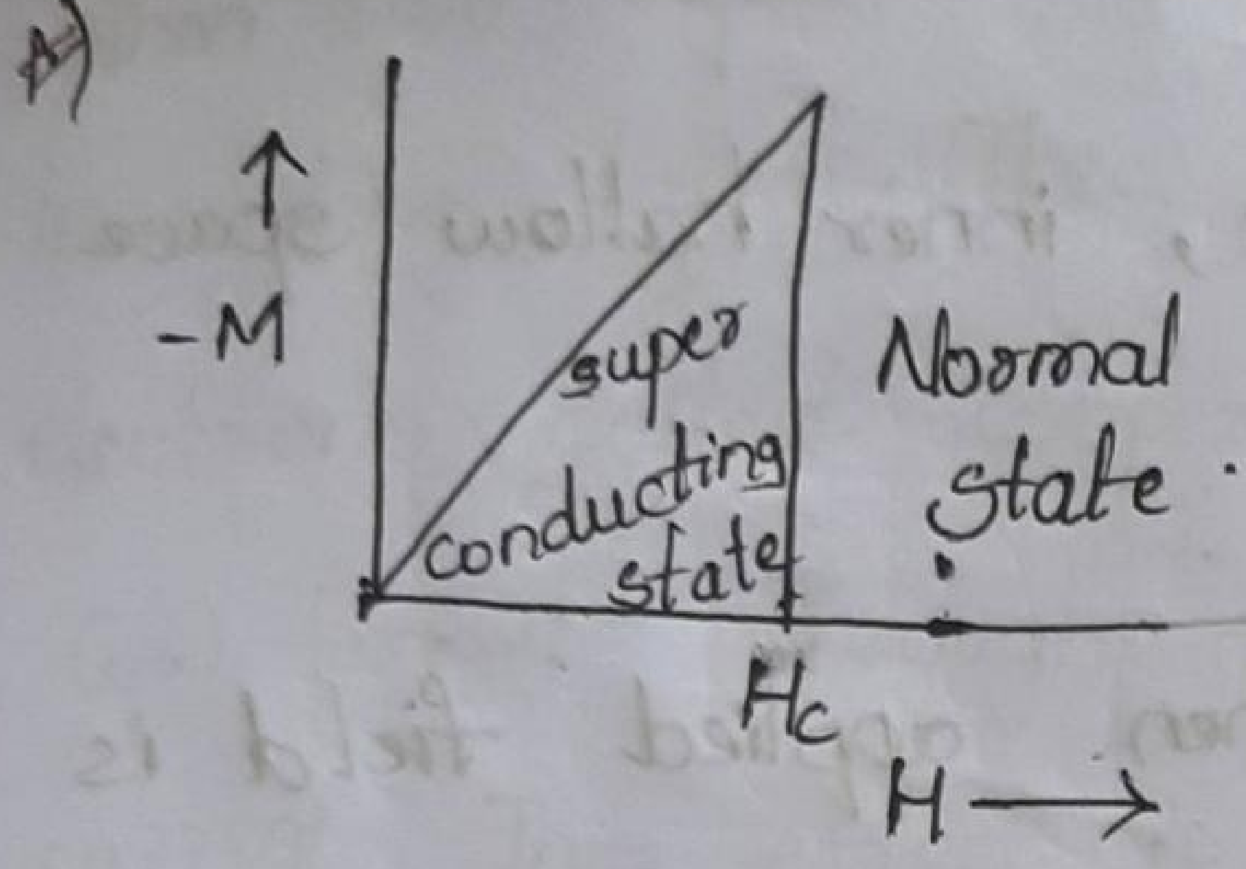
Normal
$T > T_c$ or $H > H_c$
(a)

Superconducting
$T < T_c$ or $H < H_c$
(b)

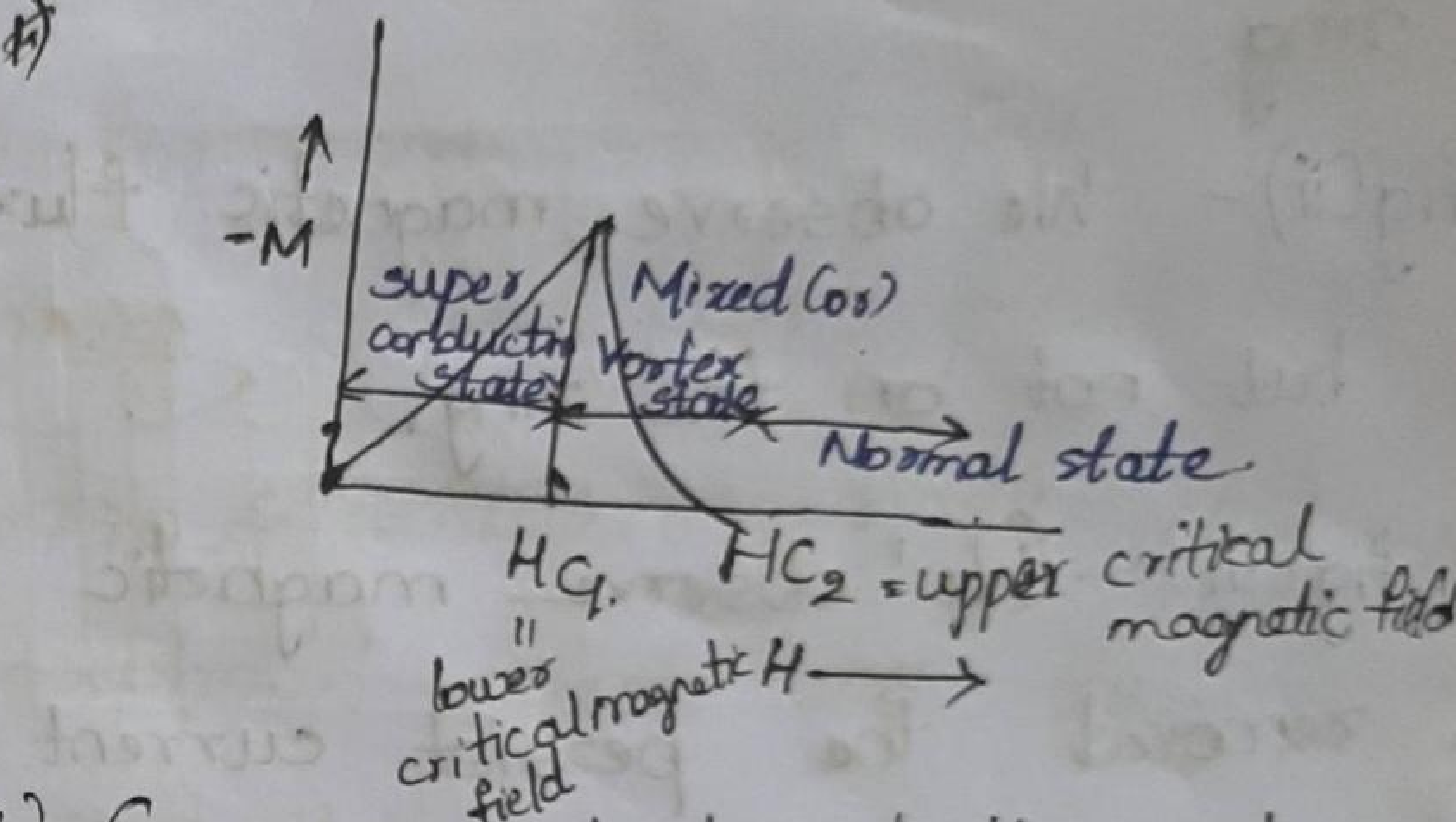
Fig. 3 In the superconducting state, the flux lines of a magnetic field are ejected out of the superconductor (b) while in normal state it does not happen

(5M) **

A)



A)



Type-I S.C	Type-II S.C.
1.) In this super conductor when magnetic field is equal to critical magnetic field (H_c) then it immeadiately converts into normal conductor.	1.) Conversion starts at H_{c_1} and ends at H_{c_2}.
2.) It has single critical field value (H_{c})	2.) It has two critical field values (H_{c_1}, H_{c_2}).
3.) There is no mixed state	3.) There is a mixed state.
4.) They are called soft Superconductors.	4.) They are called hard super conductors.
5.) Conversion process is fast.	5.) Conversion process is slow.
6.) Material with pure form are type-I superconductors.	6.) Material with impure form are type-II super conductors.
7.) Ex:- Sn, Al, Zn, Hg	Ex:- Nb, Zr.

(5M).

quantised.

9.5 JOSEPHSON EFFECT

Consider two super conductors which are joined together with the help of a thin insulating layer as shown in Fig. 9.10. These super conductors consist of paired electrons known as Cooper pairs in the super-conducting state. These Cooper pairs will try to penetrate or tunnel through the thin insulator and consistitute a small super current. The insulator which forms the junctions between super conductors is known as Josephson junction and this effect is known as Josephson effect.

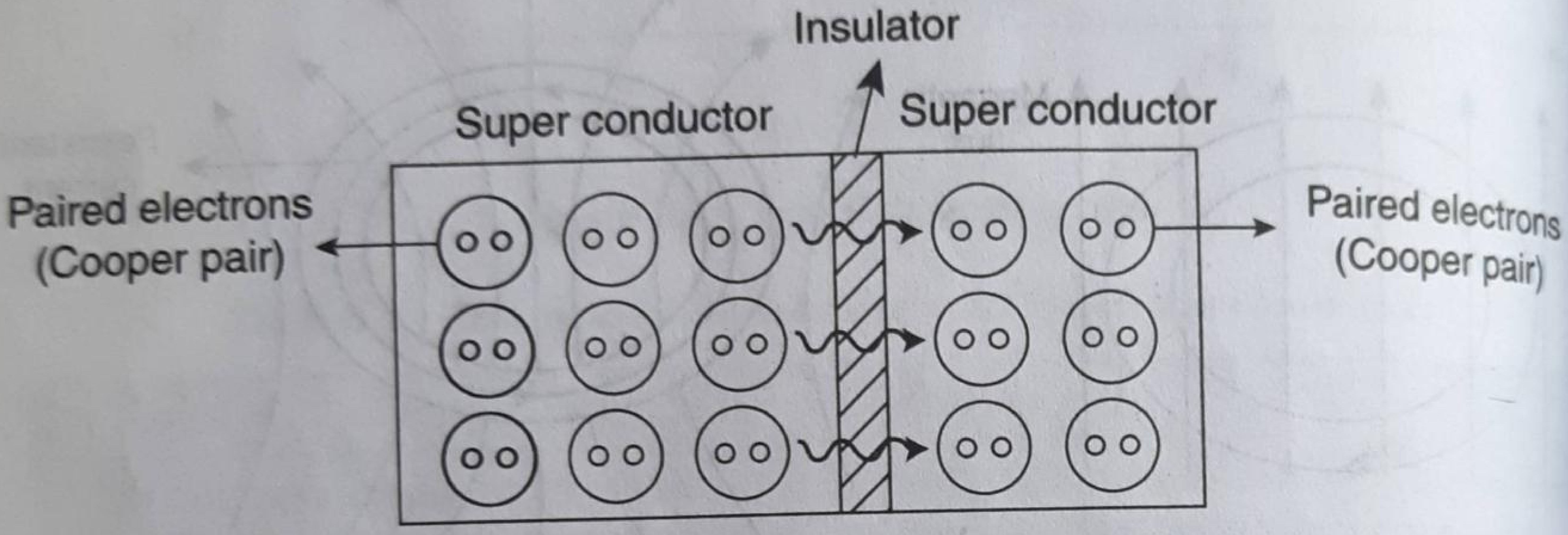


Figure 9.10 dc Josephson effect

This effect is of two types:

1. dc Josephson effect
2. ac Josephson effect

9.5.1 dc Josephson Effect

Without any applied voltage across the junction due to tunneling of Cooper pairs, a small direct super current (dc) flows across the junction. This effect is known as dc Josephson effect.

Let the propagation of Cooper pair be in the form of waves. The phase difference between the two parts of the waves on either side of the junctions in terms of wave functions is $\phi_0 = \phi_2 - \phi_1$.

The tunneling current is given by

$$I = I_0 \sin \phi_0$$

where, I_0 is the maximum current that flows through the junction without any voltage across the junction. The above expression represents a direct current (dc) that flows across the junction.

9.5.2 ac Josephson Effect

When a static potential V_0 is applied across the junction then the Cooper pairs start oscillating through the insulated layer. As a result, an alternating current (ac) flows through the junction. This effect is known as ac Josephson effect.

Due to V_0, an additional phase difference of $\Delta\phi = \frac{Et}{\hbar}$ is introduced for the Cooper pairs, where E is the total energy of the Cooper pairs at any time 't'.

$$E = (2e)V_0$$

$$\Delta\phi = \frac{2eV_0t}{\hbar}$$

The tunneling current can be written as

$$I = I_0 \sin(\phi_0 + \Delta\phi)$$

$$= I_0 \sin\left(\phi_0 + \frac{2eV_0t}{\hbar}\right)$$

$$I = I_0 \sin(\phi_0 + \omega t)$$

where $\omega = \frac{2eV_0}{\hbar}$ = angular frequency

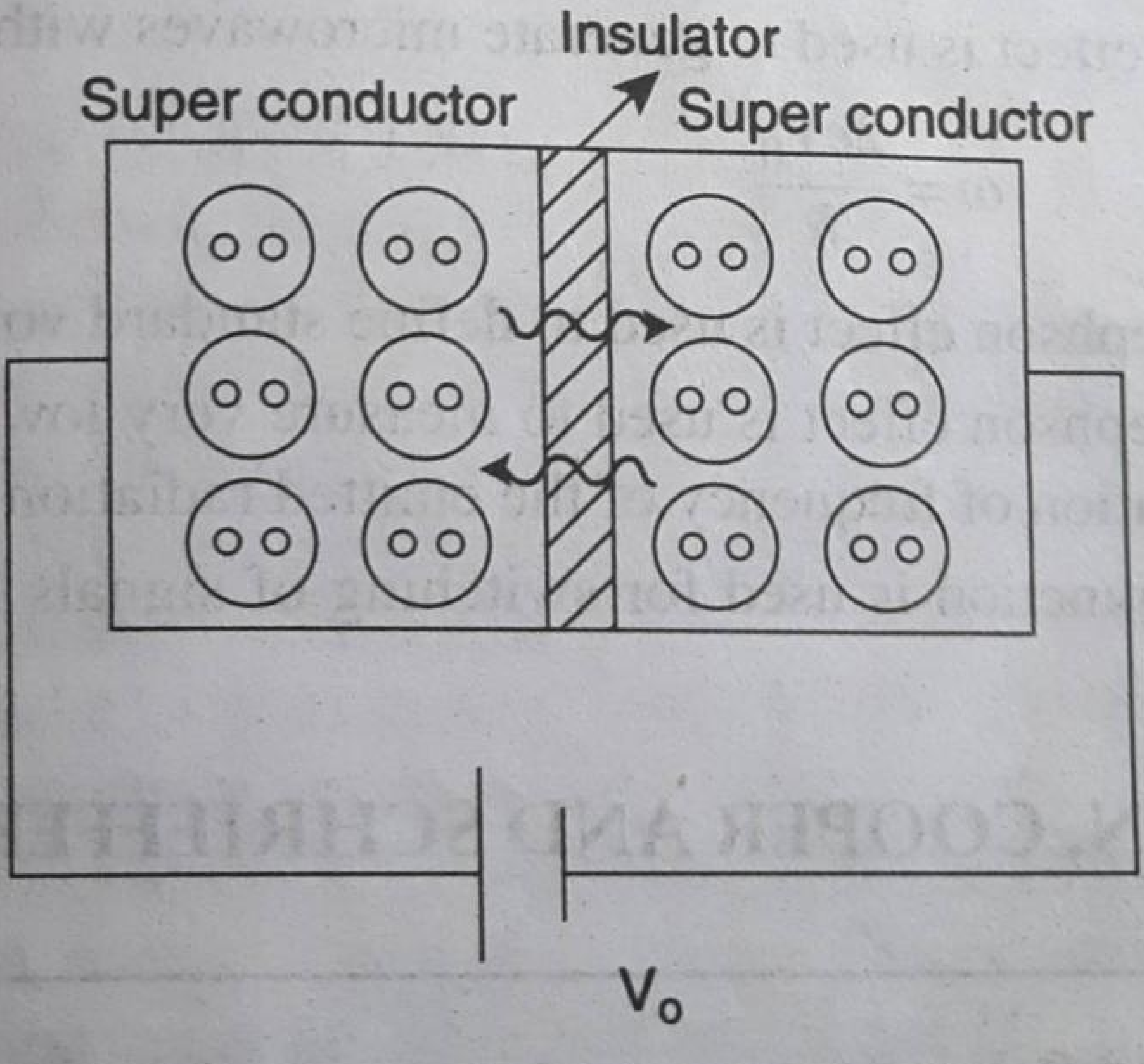


Figure 9.11 ac Josephson effect

This represents an ac with angular frequency ω. Current–voltage characteristics of a Josephson juntion is as shown in Fig. 9.12.

1. When $V_0 = 0$ there is a constant flow of dc current I_c through the junction. This current is called super conducting current and the effect is dc Josephson effect.
2. When $V_0 < V_c$, a constant dc current I_c flows.

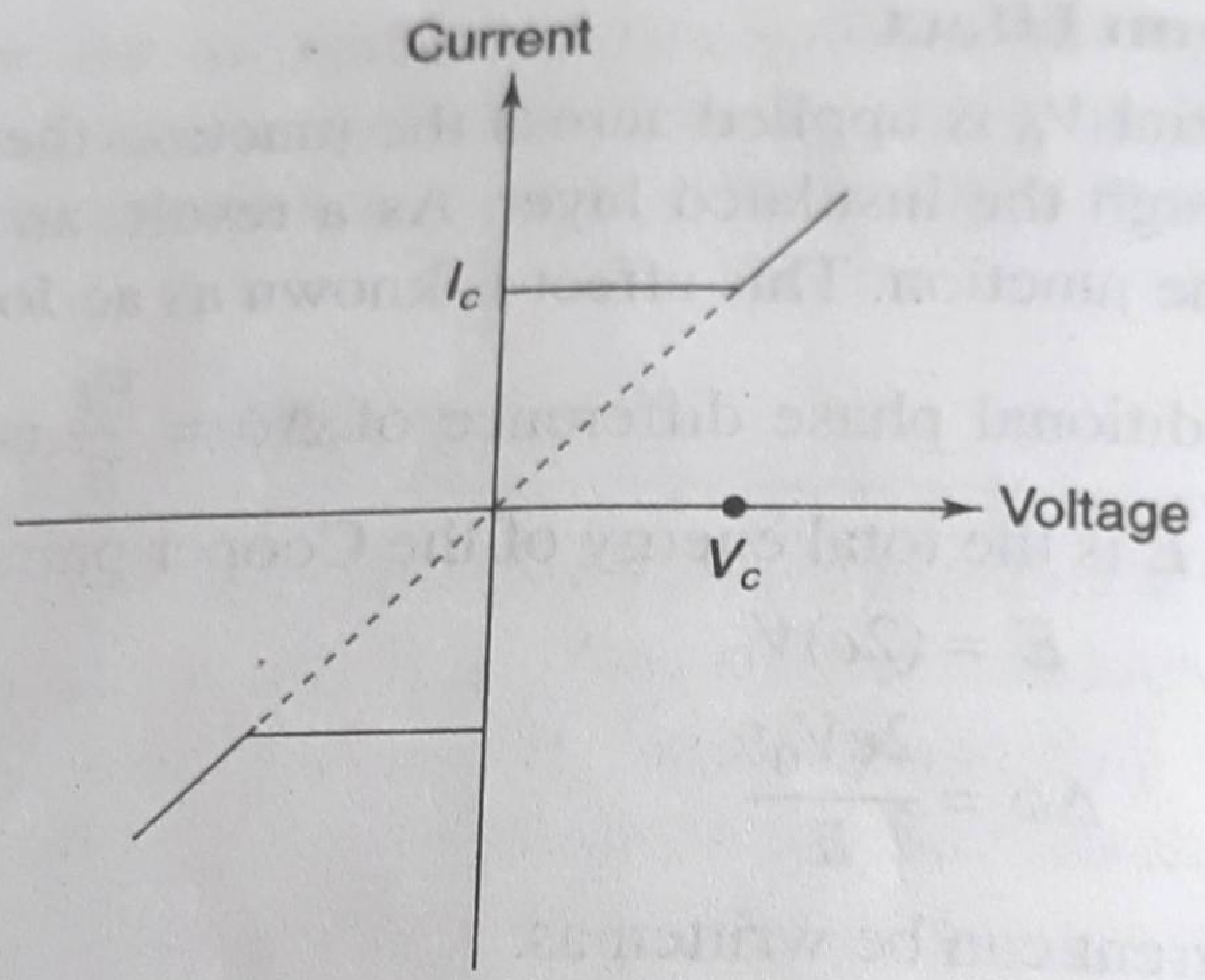


Figure 9.12 Current-voltage characteristics of Josephson junction

3. When $V_0 > V_c$, the junction has a finite resistance, and the current oscillates with frequency

$\omega = \frac{2eV_0}{\hbar}$. This effect is the ac Josephson effect.

Applications

1. Josephson effect is used to generate microwaves with frequency

$$\omega = \frac{2eV_0}{\hbar}$$

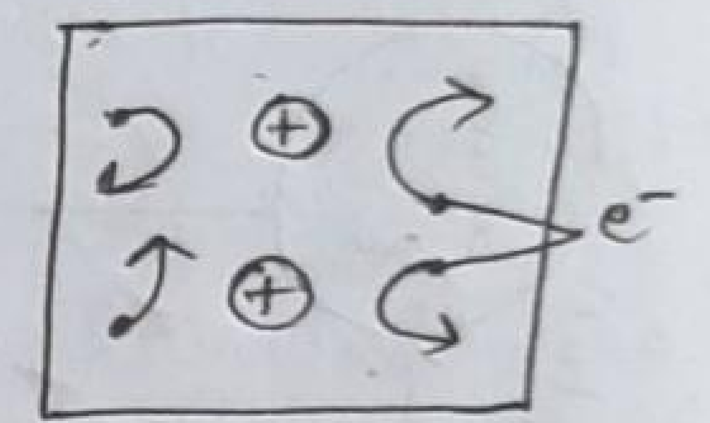
2. The ac Josephson effect is used to define standard volt.
3. The ac Josephson effect is used to measure very low temperature based on the variation of frequency of the emitted radiation with temperature.
4. Josephson junction is used for switching of signals from one circuit to another.

Bcs theory:-

Bardeen. cooper, and schrieffer proposed a Microscopic theory known as Bcs theory, which explains all the properties shown by superconductors. Such as zero resistance, Meissner effect etc;

In Normal conductor the electrons will be moving at random. when they approach vibrating atoms the repulsive force predominates than the attractive force. As a result, they get scattered and resistence comes into existence. When it is converted into super cond by decreasing its temp below the critical temp. Due to ~~decreasing~~ decrease in energy the scattering of e^- by lattice vibration also decreases, and to maintain stable state the e^- get paired up and are known as cooper pairs. The pairing of e^- can be well understood by considering the electron-phonon interaction.

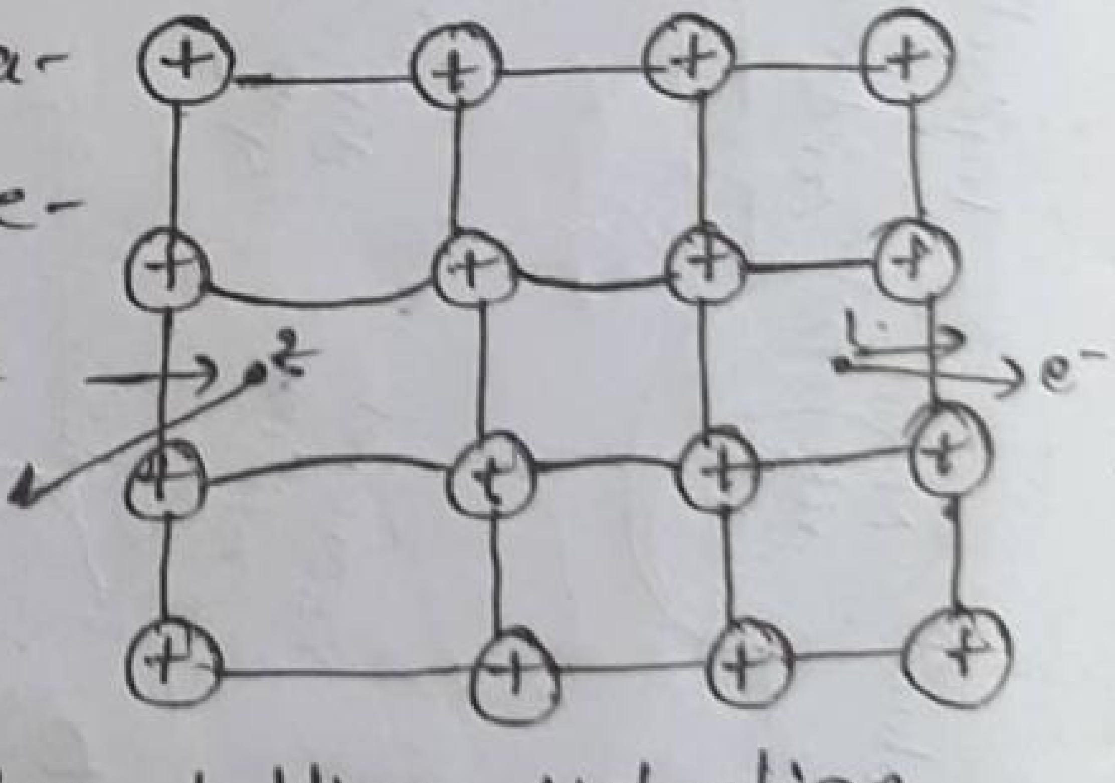
Description:-



scattering of e^- in normal conductor.

→ suppose an e^- approaches a +ve ion core in the crystal, then the e^- makes an attractive interaction with +ve ion

This attractive interaction sets in motion the +ve ion and this ion motion distorts the lattice. This distortion of lattice is quantised in terms of phonons.



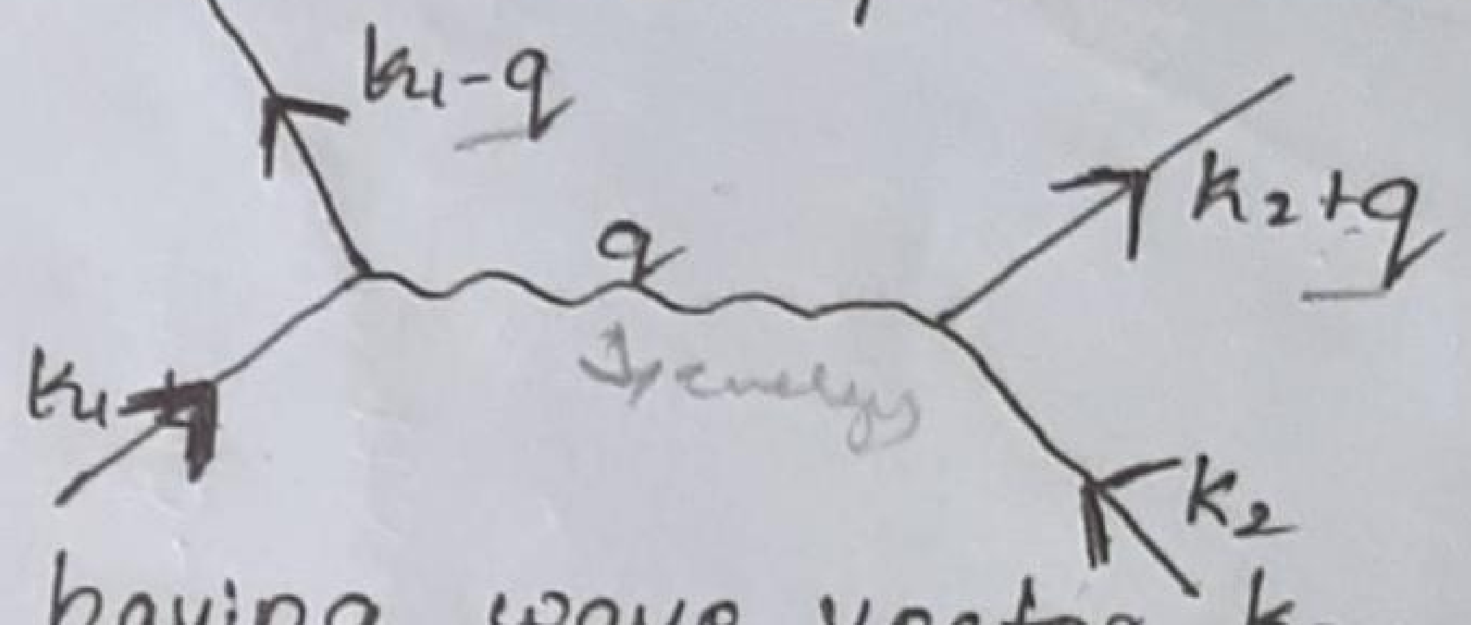
→ At that instant if another e^- approaches the distorted lattice, then the interaction b/w this 2nd e^- and distorted lattice takes place, this interaction lower the energy of 2nd e^-

→ Now the two e^- interact through the lattice distortion results in lowering of energy of the e^-. This lowering of energy indicates that an attractive force exist b/w the electrons

→ This attractive interaction is larger if the two e^- have opposite spin and momenta. This interaction is called electron ttice electron interaction (or) electron electron interaction through phonons as a mediator.

→ cooper showed that the lowering of energy leads to the formation of bound state. such bound pairs of e^- formed by the interaction b/w the e^- with opposite spin and momenta is known as cooper pairs.

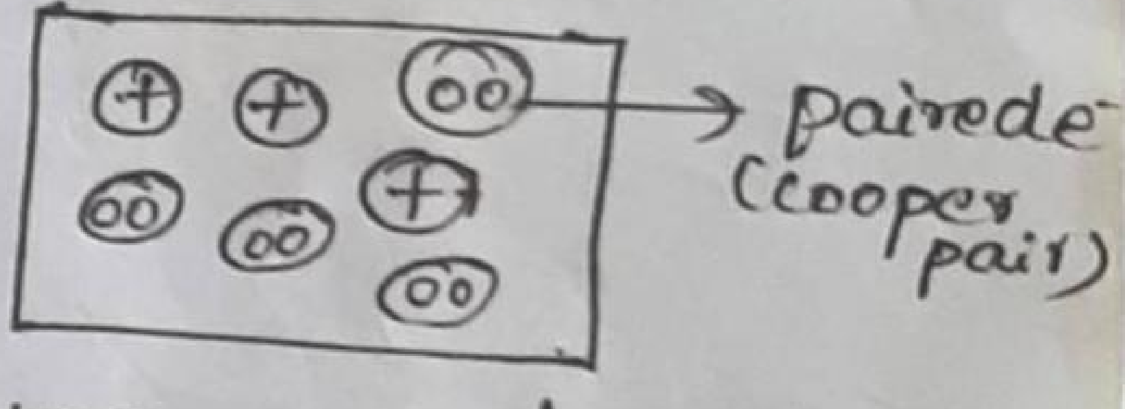
This interaction can be represented interms of the wave vector of ~~th~~ electrons.



→ let an e^- having wave vector 'k_1' emits a virtual phonon 'q' and this phonon is observed by another e^- having wave vector k_2, then k_1 is scattered as 'k_1-q' and k_2 as 'k_2+q'. conservation of energy is not satisfied in this reaction. This process is called virtual, because virtual phonons are involved in this process. the shortlived phonon is called virtual phonon.

→ A cooper pair is denoted by $(k\uparrow, k\downarrow)$

Conclusion: -



→ A cooper pair is a system of two e^- having equal and opposite momenta and spin. $(k\uparrow, k\downarrow)$

→ The energy of the pair is lowered when compared to the free seperate electrons i.e. their total energy is less than $2E_F$.

→ At lower temp (T_c) the cooper pairs interaction with +ve ion core almost vanishes and hence the resistivity becomes zero thus the conductors becomes a super conductor.

6.1 Applications of Super Conductors

1. ***Electric Generators*** Super-conducting generators are smaller in size, with less weight and consume very low energy. The low-loss superconducting coil is rotated in a strong magnetic field. This is the basis of new generation of energy-saving power systems.

2. ***Low loss transmission lines and transformers*** When super-conducting wires are used as electric cables then the transmission losses are minimised. If super-conductors are used for winding of a transformer, the power losses will be very small.
3. ***Magnetic levitation*** Diamagnetic property of a Super-conductor is the basis of magnetic levitation. This effect can be used for high-speed transportation.
4. ***Generation of high magnetic fields*** Super-conducting materials are used for producing high magnetic fields with low power consumption.
5. ***Fast electrical switching*** The application of magnetic field greater than H_c, changes the super-conducting state to normal state and removal of the field reverses the process. This principle is used in switching element cryotron.
6. ***Logic and storage functions in computers*** The C-V characteristics of Josephson effect is used for memory elements in computers. Thus, super-conductors are used to perform logic and storage functions in computers.
7. ***Super Conducting Quantum Interference Devices (SQUIDS)*** Two Josephson junctions mounted on a super conductor ring form SQUID. Since the current through SQUID is very sensitive to magnetic field, it can be used as sensitive magnetometer. These are used to study tiny magnetic signals from the brain and heart.
8. ***Super Conducting Magnets*** Super conducting magnets consists of coils of wires made up of super conductors. Current once introduced into the coil, remains for a very long time causing the stability of the magnetic field for a long time. These coils can be used in electric machines, transformers and MRI instruments.
9. ***Magnetic bearings*** Mutual repulsion between two super conducting materials due to opposite magnetic fields is used in the construction of magnetic bearings without any friction.
10. ***Super conducting sensitive magnetometer*** The quantisation of magnetic flux in SQUIDs is the basis for construction of super conducting sensitive magnetometer. It can measure magnetic field strengths of the order of 10^{-3}.
11. ***Super conducting susceptometer*** A super conducting susceptometer consists of super conducting magnet and SQUIDs. It is used to detect the variation of iron content in the human body.
12. ***Magnetoencephalography*** Doctors can locate the damaged portions of the brain by using SQUID magnetometers around the patient's head and the received magnetic signals are fed to a computer for analysis. This technique is known as magnetoencephalography.

9.6.1 High Temperature Superconductors

For most of the superconductors, superconductivity occurs only at low critical temperatures (T_C). For wide application of superconductors, scientists were trying to attain superconductivity at a much higher temperature. For attaining low temperature we should use liquid helium which is costly process. In attaining the superconductivity at high temperature or, to discover high temperature superconductors, scientists made the following progressive steps:

1. Superconductivity was discovered on a thin film of niobium and germanium at 23.2 K
2. Compound of the form Ba – PbBi–O_3 was found to be superconductor at 38 K
3. Oxide compound of the form $Y_1Ba_2Cu_3O_7$ (123 superconductor) was found to be superconductor at 92 K.
4. Oxide compound of the form Bi—Sr—Ca—Cu—O was found to be superconductor at 115 K.
5. The form Ti—Ba—Ca—Cu—O was found to be superconductor at 125 K.

In this high temperature superconductors phenomenon, liquid nitrogen (77 K) is used which is safer than liquid helium (4 K) or liquid hydrogen (23 K). It has been understood that oxygen atoms plays a major role in high temperature superconductors. Most of them have layered structure of copper and oxygen atoms.

Properties

1. They are highly anisotropic.
2. They have the presence of CuO_3 layers.
3. They have inherent metallic properties.

7.6 Isotope Effect

In superconducting materials the transition temperature varies with the average isotopic mass, M, of their constituents. The variation is found to follow the general form

$$T_c \propto M^{-\alpha} \tag{11}$$

$$\text{or} \quad M^{\alpha} T_c = \text{constant}$$

where α is called the ***isotope effect coefficient*** and is defined as

$$\alpha = \frac{\partial \ln T_c}{\partial \ln M}.$$

The value of α is approximately equal to 0.5. For example, the average isotopic mass of mercury varies from 199.5 to 203.4 atomic mass units and accordingly the transition temperature varies from 4.185 K to 4.146 K.

Solved Problem

Calculate the critical current for a wire of lead having a diameter of 1 mm at 4.2 K. The critical temperature for lead is 7.18 K and $H_o = 6.5 \times 10^4$ A/m

Solution

Formula $H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$ and $i_c = 2\pi r H_c$

$$\text{Given} \quad T = 4.2\,\text{K}$$

$$T_c = 7.18\,\text{K}$$

$$H_o = 6.5 \times 10^4\,\text{A/m}$$

$$\text{Hence} \quad H_c = 6.5 \times 10^4\left[1 - \left(\frac{4.2}{7.18}\right)^2\right]$$

$$= 4.276 \times 10^4\,\text{A/m}$$

substituting in the formula for the critical current

$$i_c = \pi \times 10^{-3} \times 4.28 \times 10^4$$

$$= 134.51\,\text{A} \qquad \text{(Answer)}$$

SOLVED PROBLEMS

1. **The critical field for niobium is 1×10^5 amp/m at 8 K and 2×10^5 amp/m at absolute zero. Find the transition temperature of the element.**

Given data Critical magnetic field at 8 K,

$$H_c = 1 \times 10^5 \text{ amp/m}$$

$$\text{Temperature } T = 8 \text{ K}$$

$$\text{Critical magnetic field at 0 K, } H_0 = 2 \times 10^5 \text{ amp/m}$$

Solution

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right] \Rightarrow \frac{H_c}{H_0} = 1-\left(\frac{T}{T_c}\right)^2$$

$$\Rightarrow \quad \left(\frac{T}{T_c}\right)^2 = 1-\frac{H_c}{H_0} \quad \text{(or)} \quad \frac{T^2}{T_c^2} = 1-\frac{H_c}{H_0}$$

$$\therefore \quad T_c^2 = \frac{T^2}{1-\dfrac{H_c}{H_0}}$$

$$\Rightarrow \quad T_c = \sqrt{\frac{T^2}{1-\dfrac{H_c}{H_0}}}$$

$$T_c = \sqrt{\frac{8^2}{1-\dfrac{1\times10^5}{2\times10^5}}} = \sqrt{\frac{64}{0.5}} = \sqrt{128} = 11.3 \text{ K}$$

2. A superconducting material has a critical temperature of 3.7 K, and a magnetic field of 0.0306 tesla at 0 K. Find the critical field at 2 K.

Given data Magnetic field at 0 K, $H_0 = 0.0306$ T

Critical temperature, $T_c = 3.7$ K

Temperature, $T = 2$ K

Solution Critical field,

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$= 0.0306\left[1-\left(\frac{2}{3.7}\right)^2\right]$$

$$= 0.0306 \times [1 - 0.2921]$$

$$= 0.0306 \times 0.7078$$

$$= 0.02166 \text{ Tesla}$$

3. If a Josephson junction has a voltage of 8.50 μV across its terminals, calculate the frequency of the alternating current. [Planck's constant = 6.626×10^{-34} J-sec]

Given data Voltage across the Josephson junction,

$$V_0 = 8.50\ \mu\text{V} = 8.5 \times 10^{-6}\ \text{V}$$

Solution Frequency of alternating current,

$$\omega = \frac{2eV_0}{h}$$

$$\upsilon = \frac{2eV}{h}$$

$$\upsilon = \frac{2 \times 1.6 \times 10^{-19} \times 8.5 \times 10^{-6}}{6.626 \times 10^{-34}} \times 27$$

$$= 1.17 \times 10^{9}\ \text{Hz.}$$