IMPORTANT QUESTIONS

CAUCHYS D.L.E :-

(i) (π) $(x^2D^2 + xD + 1)y = \log x \sin(\log x)$ → (1)

Sol Put $x = e^t$

$\log x = t$

$x\frac{dy}{dx} = Dy$; $x^2\frac{d^2y}{dx^2} = D(D-1)y$ where $D = \frac{d}{dt}$

$[D(D-1) + D + 1]y = t\sin t$

$(D^2 - \cancel{D} + \cancel{D} + 1)y = t\sin t$

$(D^2+1)y = t\sin t$

AE $m^2 + 1 = 0$

$m^2 = -1$

$m = \pm i$

$m = 0 \pm i$

CF $\cancel{C_1 \pm C_2}$ $\cancel{e^{\alpha x}(C_1 + C_2 x)}$

$e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$

$\Rightarrow e^{0x}(C_1 \cos t + C_2 \sin t)$

CF $\Rightarrow C_1 \cos t + C_2 \sin t$

$PI = \frac{1}{f(D)}\phi(x)$

$= \frac{1}{D^2+1} t\sin t$

$\boxed{e^{it} = \cos t + i\sin t}$

$\Rightarrow \frac{1}{D^2+1}$ (Imaginary part of $\frac{e^{it}}{\sin t}$) . t

$\Rightarrow$ (I.P of e^{it}) $\frac{1}{(D+i)^2+1} t$ $\left[\because \frac{1}{f(D)} e^{ax}(v) = e^{ax}\frac{1}{f(D+a)} v\right]$

$\Rightarrow$ I.P of $e^{it} \frac{1}{D^2 + 2iD + \cancel{i^2} + \cancel{1}} t$

$\Rightarrow$ I P of $e^{it} \cdot \frac{1}{2iD + D^2} \cdot t$

$\Rightarrow$ I.P of $e^{it} \cdot \frac{1}{2iD} \frac{1}{\left(1 + \frac{D^2}{2iD}\right)} t$

$\Rightarrow$ I P of $e^{it} \cdot \frac{1}{2iD}\left(\frac{1}{1 - \frac{i^2 D^2}{2iD}}\right) t$

$\Rightarrow$ I P of $e^{it} \frac{1}{2iD}\left(\frac{1}{1 - \frac{iD}{2}}\right) t$ $\Rightarrow$ I P of $e^{it} \frac{1}{2iD}\left(1 - \frac{i}{2}D\right)^{-1} t$

$\Rightarrow$ IP of $e^{it} \frac{1}{2iD}\left(1+\frac{iD}{2}\cancel{+}\right)t$

$\Rightarrow$ IP of $e^{it}\cdot\frac{1}{2iD}\left[t+\frac{i}{2}\right]$ $\left(\because (1-D)^{-1} = 1+D+D^2+D^3\right)$

$\Rightarrow$ IP of $e^{it}\cdot\frac{1}{2i}\int\left(t+\frac{i}{2}\right)dt$

$\Rightarrow$ IP of $e^{it}\ \frac{1}{2i}\left[\frac{t^2}{2}+\frac{it}{2}\right]$

$\Rightarrow$ IP of $e^{it}\left[\frac{t^2}{4i}+\frac{it}{4i}\right]$

$\Rightarrow$ IP of $e^{it}\left[\frac{-i^2t^2}{4i}+\frac{t}{4}\right]$ $\left(\because -i^2=+1\right)$

$\Rightarrow$ IP of $e^{it}\left[\frac{t}{4}-\frac{it^2}{4}\right]$

$\Rightarrow$ IP of $(\cos t + i\sin t)\left[\frac{t}{4}-\frac{it^2}{4}\right]$ $(\because i^2=-1)$

$\Rightarrow$ IP of $\left(\frac{t\cos t}{4}-\frac{it^2\cos t}{4}+\frac{it\sin t}{4}\cancel{+}-\frac{i^2t^2\sin t}{4}\right)$

$\Rightarrow$ IP of $\left(\frac{t\cos t}{4}-i\frac{t^2\cos t}{4}+\frac{it\sin t}{4}+\frac{t^2\sin t}{4}\right)$

$PI = \frac{-t^2}{4}\cos t+\frac{t}{4}\sin t$

$\therefore$ complete solution is $y = CF + PF$

$\Rightarrow C_1\cos t + C_2\sin t\ \frac{-t^2}{4}\cos t + \frac{t}{4}\sin t$

$(\because t=\log x)$

$\Rightarrow C_1\cos\log x + C_2\sin\log x - \frac{1}{4}(\log x)^2\cos\log x + \frac{1}{4}\log x\ \sin\log x$

② (A) $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$

Put $x = e^t$

$\log x = t$

$x^3D^3 = D(D-1)(D-2)y$, $x^2D^2 = D(D-1)y$; $xD = Dy$

$\therefore (D(D-1)(D-2) + 3D(D-1) + D + 1)y = e^t + t$

$\Rightarrow ((D^2 - D)(D-2) + 3D^2 - 3D + D + 1)y = e^t + t$

$\Rightarrow (D^3 - \cancel{2D^2} - \cancel{D^2} + \cancel{2D} + \cancel{3D^2} - \cancel{3D} + \cancel{D} + 1)y = e^t + t$

$\Rightarrow (D^3 + 1)y = e^t + t$

AE $= m^3 + 1 = 0$

$m = -1, \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\therefore CF = C_1 e^{-t} + e^{1/2 t}\left[C_2 \cos \frac{\sqrt{3}}{2} t + C_3 \sin \frac{\sqrt{3}}{2} t\right]$

$PI = \frac{1}{f(D)} \phi(x) \Rightarrow \frac{1}{D^3+1}(e^t + t)$

$\Rightarrow \frac{1}{D^3+1} e^t + \frac{1}{D^3+1} t$

Type ①   Type ②

Type ① $= \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$

~~Type~~

$\therefore \frac{1}{(1)^3+1} e^t + \frac{1}{(1+D^3)} t$

$\Rightarrow \frac{1}{2} e^t + \frac{1}{1}(1+D^3)^{-1} t$

$(\because (1+x)^{-1} = 1 - x + x^2 + \ldots$

$\Rightarrow \frac{1}{2} e^t + (1 - D^3 + D^6, \ ) t$

neglect $D^3, D^3, D^4, \ldots$

$\Rightarrow \frac{1}{2} e^t + (1) t$

PI $\Rightarrow \frac{1}{2} e^t + t$

$\therefore y = CF + PI$

$\Rightarrow C_1 e^{-t} + e^{1/2 t}\left[C_2 \cos \frac{\sqrt{3}}{2} t + C_3 \sin \frac{\sqrt{3}}{2} t\right]$

$+ \frac{1}{2} e^t + t$

Put $t = \log x$

$x = e^t$

$\therefore C_1 e^{-\log x} + e^{\frac{1}{2}\log x}\left[C_2 \cos \frac{\sqrt{3}}{2} \log x + C_3 \sin \frac{\sqrt{3}}{2} \log x\right]$

$+ \frac{1}{2}(x) + \log x.$

(−m)

$y = c_1 x^{-1} + x^{1/2}\left[c_2 \cos\frac{\sqrt{3}}{2}\log x + c_3 \sin\frac{\sqrt{3}}{2}\log x\right]$

$+ \frac{x}{2} + \log x.$

3(A) $(x^2D^2 + 3xD + 1)y = \log x$

Put $x = e^t$

$\log x = t$

$x^2D^2 = D(D-1)y; \quad xD = Dy$

where $D = \frac{d}{dt}$

$(D(D-1) + 3D + 1)y = t$

$(D^2 - D + 3D + 1)y = t$

$(D^2 + 2D + 1)y = t$

$AE = m^2 + 2m + 1$

$m^2 + m + m + 1$

$m(m+1) + 1(m+1)$

$(m+1)(m+1)$

$m = -1, -1$

$\therefore CF = (c_1 + c_2 x)e^{ax} \Rightarrow (c_1 + c_2 t)e^{-t}$

$PI = \frac{1}{f(D)}\phi(x) = \frac{1}{D^2 + 2D + 1} \cdot t$

$\Rightarrow \frac{1}{\left(1 + \frac{D^2 + 2D}{1}\right)} t$

$\Rightarrow \frac{1}{1}\left[1+(D^2+2D)\right]^{-1} t$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$

$\Rightarrow \left(1 - (D^2+2D) + (D^2+2D)^2 + \ldots\right] t$

$\Rightarrow \left[1 - D^2 - 2D + D^4 + 4D^2 + 4D^3 \ldots\right] t$

neglect $D^2, D^3, \ldots D^4 \ldots$

$\Rightarrow [1 - 2D] t$

$\Rightarrow t - 2Dt \qquad [D(t) = 1]$

PI $\Rightarrow t - 2$

$\therefore y = CF + PI$

$= (c_1 + c_2 t)e^{t} + t - 2$

Put $t = \log x$

$\Rightarrow (c_1 + c_2 \log x) e^{\log x} + \log x - 2$

9(A) $(x^3 D^3 + 3x^2 D^2 + xD + 8) y = 65 \cos(\log x)$ —(1)

Sol. put $x = e^t$

$\log x = t$

$x^3 D^3 = D(D-1)(D-2) y$

$3x^2 D^2 = 3D(D-1) y$

$xD = Dy$; where $D = \frac{d}{dt}$

$\therefore$ sub in (1)

$[D(D-1)(D-2) + 3D(D-1) + D + 8] y = 65 \cos t$

$\Rightarrow [(D^2 - D)(D-2) + 3D^2 - 3D + D + 8] y = 65 \cos t$

$\Rightarrow [D^3 - \cancel{2D^2} - \cancel{D^2} + \cancel{2D} + \cancel{3D^2} - \cancel{3D} + \cancel{D} + 8] y = 65 \cos t$

$\Rightarrow (D^3 + 8) y = 65 \cos t$

$\therefore AE = m^3 + 8 = 0$

$m = -2, 1 \pm i\sqrt{3}$

$\therefore CF = c_1 e^{-2t} + e^{t}[c_2 \cos\sqrt{3}t + c_3 \sin\sqrt{3}t]$

Now PI $= \frac{1}{D^2+8} 65\cos t$

$\Rightarrow 65 \frac{1}{D^2+8} \cos t$

$\left(\frac{1}{f(D)} \cos ax \quad \frac{1}{f(-a^2)} \cos ax \right)$

$\Rightarrow 65 \frac{1}{D^2 \cdot D+8} \cos t$

$\Rightarrow 65 \frac{1}{-(-1^2)D+8} \cos t = \frac{65}{-D+8} \cos D$

$\Rightarrow -65 \frac{1}{D-8} \cos D$

$\Rightarrow -65 \left[\frac{D+8}{D+8} \cdot \frac{1}{D-8} \cos D \right]$

$\Rightarrow -65 \left[\frac{D\cos D + 8\cos t}{D^2-64} \right]$

$\Rightarrow -65 \left[\frac{D\cos D + 8\cos t}{-1-64} \right] = \frac{65}{-65} D\cos D + 8\cos t$

$\Rightarrow -\sin t + 8\cos t = 8\cos t - \sin t = PI$

$\therefore y = CF + PI \Rightarrow c_1 e^{-2t} + e^t \left[c_2 \cos\sqrt{3}t + c_3 \sin\sqrt{3}t \right]$

$+ 8\cos t - \sin t$

Put $t = \log x$.

$\Rightarrow c_1 e^{-2\log x} + x\left[c_2 \cos\sqrt{3}\log x + c_3 \sin\sqrt{3}\log x \right]$

$+ 8\cos\log x - \sin\log x$

9 (B) $(x^2D^2 - xD + 2)y = x\log x$

Put $x = e^t$

$\log x = t$

$x^2D^2 = D(D-1)y$

$xD = Dy$

$\therefore (D(D-1) - D + 2)y = e^t \cdot t$

$\Rightarrow \cancel{(D^2 - D - D + 2)y = e^t \cdot t}$

$(D^2 - D - D + 2)y = t \cdot e^t$

$\Rightarrow (D^2 - 2D + 2)y = t \cdot e^t$

A.E $\Rightarrow m^2 - 2m + 2$

$\therefore m = 1 \pm i$

$\therefore CF = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$CF \Rightarrow e^t(c_1 \cos t + c_2 \sin t)$

$\therefore$ $PI = \frac{1}{f(D)} \phi(x)$

$\Rightarrow \frac{1}{(D^2-2D+2)} e^t t$

$\left[\because \frac{1}{f(D)} e^{ax} V(x) = e^{ax} \frac{1}{f(D+a)} V(x)\right]$

$\Rightarrow e^t \left[\frac{1}{(D+1)^2 - 2(D+1)+2}\right] t$

$\Rightarrow e^t \left[\frac{1}{D^2+1+\cancel{2D}-\cancel{2D}-\cancel{2}+\cancel{2}}\right] t$

$\Rightarrow e^t \left[\frac{1}{1+D^2}\right] t$

$\Rightarrow e^t [1+D^2]^{-1} t$

$\Rightarrow e^t [1 - D^2 + (D^2)^4] t$

$\left[\because (1+x)^{-1} = 1-x+x^2-x^3\right]$

neglect D^2, D^3

$\Rightarrow e^t [1] t$

$PI \Rightarrow te^t$

$\left(\text{Put } e^t = x \quad \log x = t\right)$

$\therefore y = CF + PI$

$= e^t(c_1 \cos t + c_2 \sin t) + te^t$

$\Rightarrow x(c_1 \cos\log x + c_2 \sin\log x) + x\log x$

Legendre's Linear Equation Important Sums

4 (A) $((3x+2)^2 D^2 + 3(3x+2)D - 36)y = 3x^2+4x$

Put $3x+2 = e^t$ $\quad 3x = e^t - 2 \Rightarrow x = \frac{e^t-2}{3}$

$\log(3x+2) = e^t$

$(3x+2)^2 D^2 = D(D-1)y$

$3(3x+2)D = 3Dy$

Put in (1)

$D(D-1)y + 3Dy - 36y = 3\left(\frac{e^t-2}{3}\right)^2 + 4\left(\frac{e^t-2}{3}\right)$

$\Rightarrow (D^2 - D + 3D - 36)y = \frac{\cancel{3}}{9_3}[e^{2t}+4-4e^t] + \frac{4}{3}(e^t-2)$

$\Rightarrow (D^2+2D-36)y = \frac{e^{2t}}{3} + \frac{4}{3} - \cancel{\frac{4}{3}e^t} + \cancel{\frac{4}{3}e^t} - \frac{8}{3}$

$(D^2+2D-36)y \Rightarrow \frac{e^{2t}}{3} - \frac{4}{3}$

$(D^2+2D-36)y = \frac{1}{3}(e^{2t}-4)$

$AE \Rightarrow m^2+2m-36$

$m = -1 \pm \sqrt{37}$

$\Rightarrow e^{\alpha x}(c_1 \cos\beta t + c_2 \sin\beta t)$

$C_F \Rightarrow e^{-t}\left[C_1 \cos\sqrt{37}\,t + C_2 \sin\sqrt{37}\,t\right]$

$PI = \frac{1}{f(D)}\,\phi(x)$

$= \frac{1}{D^2+2D-36}\left[\frac{1}{3}\left(e^{2t}-4\right)\right]$

$\Rightarrow \frac{1}{3}\left[\frac{1}{D^2-2D-36}\,e^{2t}\right] - \frac{4}{3}\left[\frac{1}{D^2+2D-36}\right]e^{0t}$

$\left[\frac{1}{f(a)}e^{ax}\ \frac{1}{f(a)}e^{ax}\right]$

$\Rightarrow \frac{1}{3}\left[\frac{1}{4-2(2)-36}e^{2t}\right] - \frac{4}{3}\left[\frac{1}{0+0-36}\right]$

$\Rightarrow \frac{1}{3}\left[\frac{1}{\cancel{4}-\cancel{4}-36}\cdot e^{2t}\right] - \frac{4}{3}\left[\frac{1}{-\cancel{36}_{9}}\right]$

$\Rightarrow \frac{-1}{108}e^{2t} + \frac{1}{27}$

$\Rightarrow \frac{1}{27} - \frac{1}{108}e^{2t}\ PI$

$\therefore y = CF + PI$

$\Rightarrow e^{t}\left[C_1 \cos\sqrt{37}\,t + C_2 \sin\sqrt{37}\,t\right] + \frac{1}{27} - \frac{1}{108}e^{2t}$

Put $e^{t} = 3x+2$

$\log(3x+2) = t$

$\frac{1}{(3x+2)}\left[C_1 \cos\sqrt{37}\log(3x+2) + C_2 \sin\sqrt{37}\log(3x+2)\right]$

$+\frac{1}{27} - \frac{1}{108}(3x+2)^2$

5 (A) $((1+x)^2D^2+(1+x)D+1)y = 4\cos[\log(1+x)]$ —(1)

Put $(1+x) = e^t \Rightarrow x = e^t - 1$

$t = \log(1+x)$

$\frac{d}{dt} = D$,

$\therefore (1+x)D = D$

$(1+x)^2D^2 = D(D-1)$

$(D(D-1) + D + 1)y = 4\cos t$

$\Rightarrow (D^2 \cancel{-D} + \cancel{D} + 1)y = 4\cos t$

$(D^2+1)y = 4\cos t$

$\therefore AE = m^2+1$

$m^2 = -1$

$m = 0 \pm i$

$CF = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$\Rightarrow e^{0x}(C_1 \cos t + C_2 \sin t)$

$\boxed{CF \Rightarrow C_1 \cos t + C_2 \sin t}$

$PI = \frac{1}{F(D)} \phi(x) = \frac{1}{D^2+1} 4\cos t$

$\Rightarrow 4 \frac{1}{D^2+1} \cos t$

type 2

type (2) $\Rightarrow \left[\frac{1}{f(D)} \cos ax = \frac{1}{f(-a^2)} \cos ax\right]$

$N_n = 0$

$\Rightarrow 4t \frac{1}{2D} \cos t$

$\Rightarrow {}^2\cancel{4}t \frac{1}{\cancel{2}} \int \cos t$ $\left[\because \int \sin x = -\cos x,\ \int \cos x = \sin x\right]$

$\Rightarrow 2t \sin t$

$\therefore y = CF + PI = C_1 \cos t + C_2 \sin t + 2t \sin t$

put $t = \log(1+x)$

$\Rightarrow C_1 \cos \log(1+x) + C_2 \sin \log(1+x) + 2\log(1+x) \sin \log(1+x)$

6(A) $((x+1)^2D^2 - 3(x+1)D + 4)y = x^2 + x + 1$

Put $1+x = e^t$ $\quad x = e^t - 1$

$\log(1+x) = t$

$\frac{d}{dt} = D$

$\Rightarrow \therefore (1+x)D^2 = D(D-1)y$

$-3(1+x)D = -3Dy$

$\therefore (D(D-1) - 3D + 4)y = (e^t - 1)^2 + e^t - 1 + 1$

$(D^2 - D - 3D + 4)y = e^{2t} + 1 - 2e^t + e^t$

$(D^2 - 4D + 4)y = e^{2t} - e^t + 1$

AE $= m^2 - 4m + 4$

$(m-2)(m-2)$

$m = 2, 2$

$\therefore CF = (C_1 + C_2 x)e^{2x}$

$\boxed{CF = (C_1 + C_2 t)e^{2t}}$

$PI = \frac{1}{f(D)} g(x) = \frac{1}{D^2 - 4D + 4}(e^{2t} - e^t + 1)$

$\Rightarrow \frac{1}{D^2 - 4D + 4} \cdot e^{2t} - \frac{1}{D^2 - 4D + 4} e^t + \frac{1}{D^2 - 4D + 4} e^{0t}$

$D \to a$

$\Rightarrow \frac{1}{(4) - 4(2) + 4} e^{2t} - \frac{1}{(1)^2 - 4 + 4} e^t + \frac{1}{0 - 0 + 4} e^{0t}$

$+4 - 4(2) + 4$

$\Rightarrow \frac{e^{2t}}{16} + e^t$

$\frac{1}{4D} e^{2t} - \frac{e^t}{1} + \frac{1}{4}$

$\Rightarrow \frac{1}{4} \int e^{2t} + e^t + \frac{1}{4}$

$\Rightarrow \frac{1}{4} \frac{e^{2t}}{2}$

$\Rightarrow \frac{e^{2t}}{8} + e^t + \frac{1}{4}$

typeI diff w.r.t

$\Rightarrow \frac{t}{2D - 4} e^{2t} - \frac{1}{-1 - 4 + 4} e^t + \frac{1}{0 + 0 + 4} e^{0t}$

D→0 again diff

$\boxed{PI \Rightarrow \frac{t^2}{2} e^{2t} - e^t + \frac{1}{4}}$

$\therefore y = CF + PI$

$\Rightarrow (C_1 + C_2 t) e^{2t} + \frac{t^2}{2} e^{2t} + e^{t} + \frac{1}{4}$

Put $t = \log(1+x)$, $e^t = 1+x$

$\Rightarrow (C_1 + C_2 \log(1+x)) (1+x)^2 + \frac{\log(1+x)^2}{2} (1+x)^2 + (1+x) + \frac{1}{4}$

10 (B) $((x+1)^2 D^2 + (x+1) D) y = (2x+3)(2x+4)$

$= 4x^2 + 8x + 6x + 12$

$((x+1)^2 D^2 + (x+1) D) y \Rightarrow 4x^2 + 14x + 12$

Put $1+x = e^t$ $\quad x = e^t - 1$

$\log(1+x) = t$

$D = \frac{d}{dt} \Rightarrow (x+1)^2 D^2 = D(D-1) y$

$(x+1) D = D y$

$\therefore (D(D-1) + D) y = 4(e^t - 1)^2 + 14(e^t - 1) + 12$

$(D^2 - D + D) y = 4(e^{2t} + 1 - 2e^t) + 14e^t - 14 + 2$

$D^2 y = 4e^{2t} + 4 - 8e^t + 14e^t - 12$

$D^2 y = 4e^{2t} + 6e^t - 8$

AE $= m^2 = 0$

CF $= 0$

PI $= \frac{1}{F(D)} \phi(x)$

$\Rightarrow \frac{1}{D^2} \; 4e^{2t} + 6e^t - 8$

$\Rightarrow 4 \frac{1}{D^2} e^{2t} + 6 \frac{1}{D^2} e^t - \frac{8}{D^2} e^{0t}$

type (1)

type (1) $= \frac{1}{F(D)} e^{ax} = \frac{1}{F(a)} e^{ax}$

$\Rightarrow 4 \frac{1}{(2)^2} e^{2t} + 6 \frac{1}{(1)^2} e^t - \frac{8}{0} e^{0t}$

$\Rightarrow 4 \frac{1}{4} e^{2t} + 6 \left(\frac{1}{1}\right) e^t - 0$

$\Rightarrow e^{2t} + 6e^t$

$\therefore y = CF + PI = 0 + e^{2t} + 6e^t$

Put $e^t = \log(1+x)$

$y \Rightarrow \log(1+x)^2 + 6 \log(1+x) + c$

Simultaneous linear Differential Eqn Problems Pyq

5(B): $Dx + y = \sin t$ → (1) ; $Dy + x = \cos t$ → (2)

operating by D on eqn (2)

$D^2y + Dx = D\cos t$

$D^2y + Dx = -\sin t$ → (3)

∴ (1) − (3)

$\cancel{Dx} + y = \sin t$
$D^2y + \cancel{Dx} = -\sin t$
$-\quad\quad +$

$y - D^2y = 2\sin t$

$(1 - D^2)y = 2\sin t$

A.E = $1 - m^2 = 0$

$m^2 = 1$

$m = \pm 1$

$m = 1, -1$

~~$(C_1 + C_2x)e^{dx} \Rightarrow C_1 C_2 t$~~

$\boxed{CF = C_1e^{x} + C_2e^{-x}}$

$PI = \frac{1}{F(D)}\phi(x) = \frac{1}{1-D^2} 2\sin t$

$\Rightarrow 2\frac{1}{1-D^2}\sin t$

type (2)

type (2) $\frac{1}{f(D)}\phi(x) = \frac{1}{f(-a^2)}\sin ax$ ($\sin ax$)

$\Rightarrow 2\left(\frac{1}{1-(-(1)^2)}\right)\sin t$

$\Rightarrow 2\left[\frac{1}{1+1}\right]\sin t = \frac{2\sin t}{2}$

∴ $PI = \sin t$

∴ $y = CF + PI \Rightarrow C_1e^{t} + C_2e^{-t} + \sin t$

$\boxed{y = C_1e^{t} + C_2e^{-t} + \sin t}$

from (2) $Dy + x = \cos t$

$D(C_1e^{t} + C_2e^{-t} + \sin t) + x = \cos t$
diff

$C_1e^{t} + C_2e^{-t}(-1) + \cos t + x = \cos t$

$\Rightarrow C_1e^{t} - C_2e^{-t} + \cos t + x = \cos t$

$\boxed{x = C_2e^{-t} - C_1e^{t}}$ are the solutions.

6(B) given $\frac{dx}{dt} + y = e^t$, $\frac{dy}{dt} - x = e^{-t}$

$Dx + y = e^t$; $Dy - x = e^{-t}$
$\rightarrow (1)$ $\rightarrow (2)$

operate eq-(2) with D

$D^2y - Dx = D(e^{-t})$

$D^2y - Dx = -1e^{-t}$

$D^2y - Dx = -e^{-t}$ $\rightarrow (3)$

from (1) & (3)

$\cancel{D}x + y = e^t$
$D^2y \cancel{- Dx} = -e^{-t}$

$D^2y + y = 0$

$(D^2+1)y = 0$

$m^2+1 = 0$

$m^2 = -1$

$m = 0 \pm i$

$\therefore CF = e^{0t}(C_1 \cos t + C_2 \sin t)$

$CF = C_1 \cos t + C_2 \sin t$

$PI = \frac{1}{f(D)}(g(x)) = \frac{1}{D^2+1}(0)$

$PI = 0$

$\boxed{\therefore y = C_1 \cos t + C_2 \sin t}$

$\therefore$ from (2) $Dy - x = e^{-t}$

$D(C_1 \cos t + C_2 \sin t) - x = e^{-t}$

$\rightarrow -C_1 \sin t + C_2 \cos t = e^{-t} + x$

$\boxed{C_2 \cos t - C_1 \sin t - e^{-t} = x}$

$\therefore x, y$ are the solutions.

9(A) $(D+6)y - Dx = 0$ —(1) $-Dx - 2Dy = 0$ —(2)

~~operate eqn (2) with D~~

from (1) & (2)

$(D+6)y - Dx = 0$

$(-)\quad -2Dy - Dx = 0$

$+ \qquad +$

$(D+6)y + 2Dy = 0$

$Dy + 6y + 2Dy = 0$

$3Dy + 6y = 0$

$(3D+6)y = 0$

$\therefore$ AE $= 3m+6$

$3m = -6$

$\boxed{m = -2}$ $\boxed{m = 0}$

$CF = C_1 e^{0x} + C_2 e^{-2x}$

$PI = \frac{1}{(3D+6)}(0) \Rightarrow 0.$

$\boxed{y = C_1 e^{0x} + C_2 e^{-2x}}$

$\therefore -Dx - 2Dy = 0$

$-Dx - 2D[C_1 e^{0x} + C_2 e^{-2x}] = 0$

$\Rightarrow -Dx - 2[0 - 2C_2 e^{-2x}] = 0$

$Dx = 4C_2 e^{-2x}$

$x = \frac{1}{D} 4C_2 e^{-2x}$

$x = 4C_2 \int e^{-2x}$

$x = \frac{4C_2 e^{-2x}}{-2}$

$\boxed{x = -2e^{-2x} C_2}$

$\therefore x = -2e^{-2x} C_2 ; \; y = C_1 e^{0x} + C_2 e^{-2x}$

Q(B) $\frac{\partial x}{\partial t} + 5x - 2y = t$; $\frac{\partial y}{\partial t} + 2x + y = 0$

$Dx + 5x - 2y = t$; $Dy + 2x + y = 0$

$\Rightarrow (D+5)x - 2y = t$ —(1) ; $(D+1)y + 2x = 0$ —(2)

operate $(D+5)$ on eq (2)

$$(D+5)(D+1)y + 2(D+5)x = 0$$

$$(D^2 + D + 5D + 5)y + 2(D+5)x = 0$$

$$(D^2 + 6D + 5)y + 2(D+5)x = 0 \quad \text{—(3)}$$

multiply (2) on b.s of (1)

$\Rightarrow 2(D+5)x - 4y = 2t$ —(4)

(3) & (4)

$$(D^2 + 6D + 5)y + 2(D+5)x = 0$$

$$- 4y \quad + 2(D+5)x = 2t$$

$\Rightarrow (D^2 + 6D + 5 - 4)y = -2t$

$\Rightarrow (D^2 + 6D + 1)y = -2t$

$m^2 + 6m + 1 = 0$

upon solving

$\cancel{m = \frac{-3+2\sqrt{2}}{2}}$ $m = -3+2\sqrt{2}$; $-3-2\sqrt{2}$

$\cancel{(c_1 + c_2 x)}$

$\Rightarrow c_1 e^{x} + c_2 e^{2x}$

$$\boxed{CF \Rightarrow c_1 e^{-3+2\sqrt{2}\,t} + c_2 e^{-3-2\sqrt{2}\,t}}$$

$PI = \frac{1}{D^2 + 6D + 1}(-2t) \quad -2 \frac{1}{(1)^2 + 6(1) + 1} t$

$\Rightarrow -2 \frac{1}{8_4} t$

$PI \Rightarrow -\frac{t}{4}$

$$\boxed{y = c_1 e^{-3+2\sqrt{2}\,t} + c_2 e^{-3-2\sqrt{2}\,t} - \frac{t}{4}}$$

$D(y) = -3+2\sqrt{2}t\, c_1 e^{-3+2\sqrt{2}t} - 3-2\sqrt{2}t\, e^{-3\sqrt{2}t} - \frac{1}{4}$

$\frac{Dy + y}{2} \Rightarrow x \Rightarrow x \Rightarrow \frac{-3+2\sqrt{2}c_1 e^{-3+2\sqrt{2}t} - 3-2\sqrt{2}t\, e^{-3\sqrt{2}t} - \frac{1}{4} + c_1 e^{-3+2\sqrt{2}t} + c_2 e^{-3-2\sqrt{2}} - \frac{t}{4}}{2}$

$$x = \frac{1}{2} e^{-3+2\sqrt{2}}\left[-3+2\sqrt{2}\,c_1 + c_1\right] + e^{-3-2\sqrt{2}}\left[-3-2\sqrt{2}\,c_2 + c_2\right]$$

Problems of Electrical circuit