HOMOGENEOUS LINEAR PDE IMP QUESTIONS.

(6) (A) given PDE is

$$(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$$

AE is

$$m^3 - 3m^2 + 4$$

upon solving

$\cancel{m = 2, -1, 2}$ $m = -1, 2, 2$

$\cancel{\therefore CF = f_1(y+x) + f_2(y-x) + x f_3(y)}$

$$CF = f_1(y-x) + f_2(y+2x) + x f_3(y+2x)$$

$$PI = \frac{1}{f(D, D')} F(x, y)$$

$$\Rightarrow \frac{1}{D^3 - 3D^2D' + 4D'^3} \cdot e^{x+2y}$$

e^{ax+by}

Put $D = 1;\ D' = 2$ $a = 1, b = 2$

$$\therefore \frac{1}{(1)^3 - 3(1)^2(2) + 4(2)^3}$$

$$PI \Rightarrow \frac{1}{1 - 6 + 32} e^{x+2y} = \frac{e^{x+2y}}{27}$$

$y = CF + PI$

$= f_1(y0 - x) + f_2(y+2x) + x f_3(y+2x)$

$+ \frac{1}{27} \cdot e^{x+2y}$

6 (B) $(D^3 - 4D^2D' + 4DD'^2)z = 2\sin(3x+2y)$

$AE: m^3 - 4m^2 + 4m + 0 = 0$

$m = 2, 2, 0$

$\therefore CF = f_1(y + 0x) + f_2(y+2x) + x f_3(y+2x)$

$CF = f_1(y) + f_2(y+2x) + x f_3(y+2x)$

$PI = \frac{1}{f(D, D')} 2\sin(3x+2y)$

$\Rightarrow \frac{1}{D^3 - 4D^2D' + 4DD'^2} 2\sin(3x+2y)$

$\sin mx + ny$

$\frac{1}{D^2 \cdot D - 4D^2D' + 4DD'^2} \cdot \sin(3x+2y)$

$D^2 = -m^2 = -9$, $D'^2 = -n^2 = -4$

$DD' = -(3)(2) = -6$

$4D^2D' = 4(DD')(D)$

$\Rightarrow \frac{1}{-9D - 4(-6)D + 4D(-4)} 2\sin(3x+2y)$

$\Rightarrow \frac{1}{-9D + 24D - 16D} 2\sin(3x+2y)$

$\Rightarrow \frac{1}{-D} 2\sin(3x+2y)$

$\Rightarrow -2\left[\frac{1}{D}\sin(3x+2y)\right]$

$\Rightarrow -2\int \sin(3x+2y)$

$\Rightarrow -2\left[\frac{-\cos(3x+2y)}{3}\right]$

$PI \Rightarrow \frac{+2}{3}\cos(3x+2y)$

$y = CF + PI = f_1(y) + f_2(y+2x) + x f_3(y+2x)$

$+ \frac{2}{3}\cos(3x+2y)$

(7) $(D^3 - 7DD'^2 - 6D'^3) = \sin(x+2y) + e^{2x+y}$

AE is

$m^3 - 7m - 6$

upon solving $m = -1, -2, 3$

$\therefore$ CF $= f_1(y-x) + f_2(y-2x) + f_3(y+3x)$

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \cdot \sin(x+2y) + \frac{1}{D^3 - 7DD'^2 - 6D'^3} e^{2x+y}$$

$\downarrow$ (1) $\qquad$ (2)

$\Rightarrow$ from (1)

$$\frac{1}{D^2 D - 7D \cdot D'^2 - 6D'^2 \cdot D'} \cdot \sin(x+2y)$$

$D^2 = -1$

$D'^2 = -4$

$DD' = -(1)(2) = -2$

$$\Rightarrow \frac{1}{(-1)D - 7D(-4) - 6D'(-4)} \cdot \sin(x+2y)$$

$$\frac{1}{-D + 28D + 24D'} \cdot \sin(x+2y)$$

$$\Rightarrow \frac{1}{27D + 24D'} \sin(x+2y)$$

$$\Rightarrow \frac{1}{27D + 24D'} \times \frac{(27D - 24D')}{(27D - 24D')} \cdot \sin(x+2y)$$

$$\Rightarrow \frac{(27D - 24D') \sin(x+2y)}{729D^2 - 576D'^2}$$ So

$D^2 = -1$; $D'^2 = -4$

$$\Rightarrow \frac{(27D - 24D') \sin(x+2y)}{729(-1) - 576(-4)}$$

$$\Rightarrow \frac{1}{1575} [27D \sin(x+2y) - 24D' \sin(x+2y)]$$

$\because D = \frac{d}{dx}$; $D' = \frac{d}{dy}$

$$\Rightarrow \frac{1}{1575} [27 \cos(x+2y) - 24 \cos(x+2y)(2)]$$

$$PI = \frac{1}{1575}\left[27\cos[x+2y] - 48\cos(x+2y)\right]$$

$$\Rightarrow \frac{1}{\cancel{1575}_{75}}\left[-\cancel{21}\cos(x+2y)\right]$$

$$PI = \frac{-1}{75}\cos(x+2y)$$

$$y = CF + (PI_1) + (PI)_2$$

$$= f_1(y-x) + f_2(y-2x) + f_3(y+3x)$$

$$\frac{-1}{75}\cos(x+2y) \rightarrow (2)$$

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consider $\frac{1}{D^3 - 7DD'^2 - 6D'^3} \cdot e^{2x+y}$ $\quad e^{ax+by}$

$(PI)_2$

$\cancel{D=a, D'}$ $\quad D = a = 2$

$D' = b = 1$

$$\Rightarrow \frac{1}{8-(7)(2)(1)-6(1)} e^{2x+y} = \frac{1}{8-14-6} e^{2x+y}$$

$$\Rightarrow \frac{1}{-12} e^{2x+y} \rightarrow (3)$$

from (2) & (3)

$$\therefore y = f_1(y-x) + f_2(y-2x) + f_3(y+3x)$$

$$\frac{-1}{75}\cos(x+2y)\ \frac{-1}{12}e^{2x+y}.$$

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(8) $(D^2 + DD' - 6D'^2) = x+y$

AE is

$m^2 + m - 6$

$(m+3)(m-2)$

$m = 2, -3$

$$CF = f_1(y+2x) + f_2(y-3x)$$

$$PI = \frac{1}{D^2 + DD' - 6D'^2} (x+y)$$

$$\Rightarrow \frac{1}{(D+3D')(D-2D')}(x+y)$$

$$\Rightarrow \frac{1}{D+3D'}\left[\frac{1}{(D-2D')}(x+y)\right]$$

comp $m=2$

$D - mD'$

~~$y = mx + c$~~

$y = c - mx$

$y = c - 2x$

$$\Rightarrow \frac{1}{D+3D'} \int (x + c - 2x)\, dx$$

$= \frac{1}{D+3D'} \int (c-x)\,dx$

$\Rightarrow \frac{1}{D+3D'} \left[c\int dx - \int x\,dx\right]$

$\Rightarrow \frac{1}{D+3D'} \; c(x) - \frac{x^2}{2}$

$\Rightarrow \frac{1}{D+3D'} \left(c\cancel{x} - \frac{x^2}{2}\right]$

$y = c - mx$

$c = y + mx$

$c = y + 2x$

$\Rightarrow \frac{1}{D+3D'} \left[(y+2x)\,x - \frac{x^2}{2}\right]$

$\Rightarrow \frac{1}{D+3D'} \left[xy + \frac{2x^2}{1} - \frac{x^2}{2}\right]$

$2x^2 - \frac{x^2}{2}$

$\Rightarrow \frac{4x^2 - x^2}{2} = \frac{3x^2}{2}$

$\Rightarrow \frac{1}{D+3D'} \left[xy + \frac{3x^2}{2}\right]$

comp with ~~D-m~~ $D - mD'$

$m = -3$

$y = c_1 - mx$

$= c_1 - (-3)x = c + 3x$

$\int x(c_1 + 3x) + \frac{3x^2}{2}$

$\int c_1 x + 3x^2 + \frac{3x^2}{2}$

$\Rightarrow c_1 \frac{x^2}{2} + \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{3}}{2}\frac{x^3}{\cancel{3}}$

$\Rightarrow (y-3x)\frac{x^2}{2} + x^3 + \frac{x^3}{2}$ $\quad \boxed{c_1 = y - 3x}$

$\Rightarrow \frac{x^2y - 3x^3 + 2x^3 + x^3}{2}$

$PI \Rightarrow \frac{x^2y}{2}$

$\therefore \cancel{y} = CF + PI$

$= f_1(y+2x) + f_2(y-3x) + \frac{1}{2}x^2y$

(9) $4r+12s+9t = e^{3x-2y}$

$(4D^2+12DD'+9D'^2) = e^{3x-2y}$

where $D = \frac{\partial}{\partial x}$; $D' = \frac{\partial}{\partial y}$

$\Rightarrow$ ~~$4m^2+12m+9$~~

$4m^2+12m+9$

$x = -\frac{3}{2}, -\frac{3}{2}$

$CF = f_1(y-\frac{3}{2}x) + xf_2(y-\frac{3}{2}x)$

$PI = \frac{1}{4D^2+12DD'+9D'^2} \cdot e^{3x-2y}$

$a=3;\ b=2$

$\frac{1}{4(3)^2+12(3)(-2)+9(-2)^2}$

$\Rightarrow \frac{1}{36-72+36} \cdot e^{3x-2y} = 0$

$DN=0$; diff

$= x \frac{e^{3x-2y}}{8D+12D'}$

$\frac{1}{72}$

9×4
36

$\Rightarrow \frac{x \cdot e^{3x-2y}}{8(3)+12(-2)} = 0$

$\therefore DN=0$

diff

$PI = \frac{x^2}{8} \cdot e^{3x-2y}$

$\therefore y = CF + PI$

$= f_1(y-\frac{3}{2}x) + xf_2(y-\frac{3}{2}x) + \frac{x^2}{8} e^{3x-2y}.$

(10.) $r+s-6t = y\cos x$

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\left(\frac{\partial^2 z}{\partial y^2}\right) = y\cos x$

$D^2+DD'-6D'^2 = y\cos x$

$AE = m^2+m-6$

$(m+3)(m-2)$

$\Rightarrow m = -3, 2$

$CF = f_1(y-3x) + f_2(y+2x)$

$PI = \frac{1}{F(D,D')} f(x,y)$

$\Rightarrow \frac{1}{D^2+DD'-6D'^2} y\cos x$

$\Rightarrow \frac{1}{(D+3D')(D-2D')} y\cos x$

$\Rightarrow \frac{1}{(D+3D')}\left[\frac{1}{(D-2D')} y\cos x\right]$

cmp $D-mD'$ $\underline{m=2}$

$\Rightarrow \frac{1}{D+3D'}\int (c-2x)\cos x$

$\begin{bmatrix} y = c-mx \\ y = c-2x \end{bmatrix}$

$\Rightarrow \frac{1}{D+3D'}\int c\cos x\,dx - \int 2x\cos x\,dx$

$\begin{bmatrix} c = y+mx \\ c = y+2x \end{bmatrix}$

$\Rightarrow \frac{1}{D+3D'}\left[c\sin x\right] - \left[2x\sin x - (2)(-\cos x)\right]$

$\Rightarrow \frac{1}{D+3D'}\left[(y+2x)\sin x - 2x\sin x - 2\cos x\right]$

$\Rightarrow \frac{1}{D+3D'}\left[y\sin x + \cancel{2x\sin x} - \cancel{2x\sin x} - 2\cos x\right]$

$\Rightarrow \frac{1}{D+3D'}\left[y\sin x - 2\cos x\right]$

comp with

$\frac{1}{D+mD'} f(x,y)$

$y = c-mx$

$D-mD'$

$\underline{m=-3}$

$y = c+3x$

$\int (c+3x)\sin x - 2\int \cos x$

$\Rightarrow (c+3x)(-\cos x) - (0+3)(-\sin x) - 2\sin x$

$(y-\cancel{3x}+\cancel{3x})(-\cos x) + 3\sin x - 2\sin x \quad [c = y-3x]$

$PI = -y\cos x + \sin x$

$y = CF + PI = f_1(y-3x) + f_2(y+2x) + \sin x - y\cos x$

Legendre linear PDE imp problems.

(2) (B) $y^2p - xyq = x(z-2y)$

$Pp + Qq = R$

AE is

$\therefore \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$

First 2 fractions

$\frac{dx}{y^{\cancel{2}}} = \frac{dy}{-x\cancel{y}}$

$\frac{dx}{y} = \frac{dy}{-x}$

$-x\,dx = y\,dy$

'J' in b-s

$-\frac{x^2}{2} = \frac{y^2}{2} + c_1$

$\frac{x^2}{2} + \frac{y^2}{2} = c_1$

$x^2 + y^2 = c_1 = 2c_2$

$x^2 + y^2 = 2c_2$ ---> (4)

from last 2 fractions

$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$

$(z-2y)dy = -y\,dz$

$z\,dy - 2y\,dy = -y\,dz$

~~$-y\,dz - 2y\,dy = z\,dy$~~

$z\,dy + y\,dz = 2y\,dy$

$d(zy) = 2y\,dy$

'J' in b-s

$zy = \frac{2y^2}{2} + c_3$

$c_3 = zy - y^2$ ---> (V)

Solution is $f(u,v)=0$

$$F(x^2+y^2, zy-y^2)=0$$

(3)(B) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

AE are

$$\frac{dx}{x^2(y-z)}=\frac{dy}{y^2(z-x)}=\frac{dz}{z^2(x-y)}$$

using $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are multiplier.

$$\frac{dx/x}{x(y-z)}=\frac{dy/y}{y(z-x)}=\frac{dz/z}{z(x-y)}$$

$$\frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{0}=0$$

$$\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}=0$$

$\int$ on b.s

$$\log x+\log y+\log z=\log c_1$$

$$\log(xyz)=\log c_1$$

$$\boxed{xyz=c_1} \rightarrow (4)$$

using $\frac{1}{x^2},\frac{1}{y^2},\frac{1}{z^2}$ as multipliers

$$\frac{dx/x^2}{y-z}=\frac{dy/y^2}{z-x}=\frac{dz/z^2}{x-y}$$

$$\frac{dx}{x^2}+\frac{dy}{y^2}+\frac{dz}{z^2}=0$$

$\int$ on b.s

$$\left(\frac{-1}{x}\right)+\left(\frac{-1}{y}\right)+\left(\frac{-1}{z}\right)=c_2$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-c_2=c_3 \rightarrow (v)$$

$$f(u,v)=0=F\left(xyz, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

4(A) $p^2+pq=z^2$

$pp+pq=z^2$

$$\frac{dx}{p}=\frac{dy}{p}=\frac{dz}{z^2}$$

from first 2 fractions

$$\frac{dx}{p}=\frac{dy}{p}$$

'∫' on b-s

$\int dx=\int dy$

$\Rightarrow x=y+c$

$x-y=c_1 \rightarrow (4)$

from last (2) fractions

$$\frac{dy}{p}=\frac{dz}{z^2}$$

$z^2dy=p\,dz$

1(B) $(y-z)p+(x-y)q=z-x$

$$\frac{dx}{(y-z)}=\frac{dy}{(x-y)}=\frac{dz}{z-x}$$

1, 1, 1 are multipliers

$$\frac{dx+dy+dz}{y-z+x-y+z-x}=0$$

$dx+dy+dz=0$

∫ on b-s

$\int dx+\int dy+\int dz=c_1$

$x+y+z=c_1 \rightarrow (4)$

~~$xy-xz+yx-y^2+z^2-$~~

~~$x-xz+xz-zy+yz-xy$~~

$=0$

$$\frac{dx+dy+dz}{y-z+x-y+z-x}=0$$

$=0$

$\int dx+\int dy+\int dz=\int 0$

$x+y+z=c_1 \Rightarrow (4)$

multipliers are x, y, z

$$\frac{x\,dx}{x(y-z)}=\frac{z\,dy}{z(x-y)}=\frac{y\,dz}{y(z-x)}$$

$x\,dx + z\,dy + y\,dz = 0$

$\Rightarrow \frac{x^2}{2} + \frac{z^2}{2} + \frac{y^2}{2} = c_2$

$x^2 + y^2 + z^2 = 2c_2 = c_3 \rightarrow (v)$

$\Rightarrow F(u,v) = F(x+y+z,\ x^2+y^2+z^2) = 0$

Non-Linear Type:-

4(A) $p^2 + pq = z^2 \rightarrow (1)$

this is non linear PDE

eqn (1) contains z, p, q and not involving PDE

eqn (1) is of form $f(z, p, q) = 0$

Put $q = ap \rightarrow (2)$

$p^2 + p(ap) = z^2$

$p^2 + ap^2 = z^2$

$(1+a)p^2 = z^2$

from this we can obtain in terms of z

$p^2 = \frac{z^2}{1+a}$

$\boxed{p = \frac{z}{\sqrt{1+a}}}$

Sub p in (2)

$q = ap = a\left(\frac{z}{\sqrt{1+a}}\right)$

$\boxed{q = \frac{az}{\sqrt{1+a}}}$

Sub p, q values in $\boxed{dz = p\,dx + q\,dy}$

$$dz = \frac{z}{\sqrt{1+a}}\,dx + \frac{a \cdot z}{\sqrt{1+a}}\,dy$$

$$dz = \frac{z}{\sqrt{1+a}}\left[dx + a\,dy\right]$$

$$\frac{dz}{z} = \frac{1}{\sqrt{1+a}}(dx + a\,dy)$$

'∫' on b.s

$$\int \frac{dz}{z} = \frac{1}{\sqrt{1+a}}\left(\int dx + a\int dy\right)$$

$$\log z = \frac{1}{\sqrt{1+a}}(x + ay) + c$$

which is req. ~~soln~~ complete solutions of eqn ① containing 2 arbitrary constants a & c.

(5) ~~$z^2 = px^2 + qy$~~

(B) $xp - yq = x^2 + y^2$ is Legendre L PDE

Put $z = x^2 + y^2$

AE is $\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{z}$

$$\frac{dx}{x} = \frac{dy}{-y}$$

$$-y\,dx = x\,dy$$

∫ on b.s

$-\frac{y^2}{2}$

$$\log x = -\log y + c$$

$$\log x + \log y = \log c$$

$$\cancel{\log}(xy) = c_1 \Rightarrow (u)$$

$$\frac{dy}{-y} = \frac{dz}{z}$$

$$-\log y = \log z + \log c$$

$$\log z + \log y = \log c_2$$

$$zy = c_2 \Rightarrow (v)$$

$f(u, v) = 0 \Rightarrow f(xy, yz) = 0$

$f(xy, y(x^2+y^2)) = 0$ //

Elimination of arbitrary constants / functions

(2) (A) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ → (1)

partially diff w.r.t 'x',

$$\frac{\partial}{\partial x}\left[\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right]=\frac{\partial}{\partial x}(1)$$

(∵ y is constant)

$$\Rightarrow \frac{1}{a^2}(2x)+0+\frac{1}{c^2}\cdot 2z\frac{\partial z}{\partial x}=0$$

$$\Rightarrow 2\left[\frac{x}{a^2}+\frac{z}{c^2}\cdot\frac{\partial z}{\partial x}\right]=0$$

$$\frac{x}{a^2}+\frac{z}{c^2}\cdot p=0$$

→ (2)

diff w.r.t 'y'

$$\frac{\partial}{\partial x}\left[\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right]=\frac{\partial}{\partial x}(a)$$

(∵ x is constant)

$$\Rightarrow 0+\frac{1}{b^2}(2y)+\frac{2z}{c^2}\frac{\partial z}{\partial y}=0$$

$$2\left[\frac{y}{b^2}+\frac{z}{c^2}q\right]=0$$

$$\frac{y}{b^2}+\frac{z}{c^2}q=0$$

Diff eqn (2) w.r.t 'x', we get

$$\frac{\partial}{\partial x}\left[\frac{x}{a^2}+\frac{z}{c^2}\right]=0$$

$$\frac{1}{a^2}+\frac{1}{c^2}\left[z\cdot\frac{\partial}{\partial x}p+p\cdot\frac{\partial}{\partial x}z\right]=0$$

$$\frac{1}{a^2}+\frac{1}{c^2}\left[z\cdot\frac{\partial p}{\partial x}+p\cdot p\right]=0 \quad \left(\because \frac{\partial z}{\partial x}=p\right)$$

$$\frac{1}{a^2}+\frac{1}{c^2}\left[z\cdot\frac{\partial p}{\partial x}+p^2\right]=0$$

→ (4)

Multiplying on b.s eqn by 'x', we get

$$\frac{x}{a^2}+\frac{x}{c^2}\left[z\frac{\partial p}{\partial x}+p^2\right]=0 \quad (5)$$

(5) − (2)

$$\left[\frac{x}{a^2}+\frac{x}{c^2}\left(z\frac{\partial p}{\partial x}+p^2\right)-\left(\frac{x}{a^2}+\frac{zp}{c^2}\right)\right]=0$$

$=0-0.$

$$\frac{x}{c^2}\left[z\frac{\partial p}{\partial x}+p^2\right]-\frac{zp}{c^2}=0$$

$$\frac{1}{c^2}\left[xz\frac{\partial p}{\partial x}+xp^2-zp\right]=0$$

$$xz\frac{\partial p}{\partial x}+xp^2-zp=0$$

$$xzr+xp^2-zp=0$$

$$pz=xzr+xp^2$$

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(3)(A) $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$

Partially diff with w.r.t 'x'

$\left(\because p=\frac{\partial z}{\partial x}\right)$

$$\cancel{2}(x-a)+0=\cancel{2}z\cot^2\alpha\left(\frac{\partial z}{\partial x}\right)$$

$$(x-a)=z\cot^2\alpha\, p$$

Partially diff w.r.t y

$$0+\cancel{2}(y-b)=\cancel{2}z\cdot\frac{\partial z}{\partial y}\cot^2\alpha$$

$$(y-b)=zq\cot^2\alpha$$

$$(zp\cot^2\alpha)^2+(zq\cot^2\alpha)^2=z^2\cot^2\alpha$$

$$z^2p^2\cot^4\alpha+z^2q^2\cot^4\alpha=z^2\cot^2\alpha$$

$$\cancel{z^2\cot^2\alpha}\,((p^2+q^2)\cot^2\alpha=z^2\cancel{\cot^2\alpha}$$

$$\cancel{p^2+0}\quad p^2\cot^2\alpha+q^2\cot^2\alpha=1$$

$$\cot^2\alpha\,(p^2+q^2)=1$$

$$p^2+q^2=\frac{1}{\cot^2\alpha}$$

$$p^2+q^2=\tan^2\alpha$$

3(A) $\phi(x+y+z,\ x^2+y^2+z^2)=0$ $\to$ (1).

$u = x+y+z$

$v = x^2+y^2+z^2$

$\phi(u,v)=0$

it can be written as

$(u=\phi/v$ (or) $v = \phi/u)$

$x+y+z = \phi/x^2+y^2+z^2$ $\to$ (2)

eqn (2) contains only one arbitrary function ϕ, then we get a 1st order PDE

diff (2) partially w.r.t x and y; we get

$$\frac{\partial}{\partial x}(x+y+z) = \frac{\partial}{\partial x}\left[\frac{\phi}{x^2+y^2+z^2}\right]$$

($\because$ y is constant)

$$1+0+\frac{\partial z}{\partial x} = \phi'[x^2+y^2+z^2]\left(\frac{\partial}{\partial x}(x^2+y^2+z^2)\right)$$

$$1+0+p = \phi'[x^2+y^2+z^2]\left(2x+0+2z\frac{\partial z}{\partial x}\right)$$

$$1+p = \phi'[x^2+y^2+z^2](2x+2zp) \to (3)$$

diff (2) w.r.t y

$$\frac{\partial}{\partial y}[x+y+z] = \frac{\partial}{\partial y}[\phi/x^2+y^2+z^2]$$

$$0+1+\frac{\partial z}{\partial y} = \phi'(x^2+y^2+z^2)\frac{\partial}{\partial y}(x^2+y^2+z^2)$$

$\left(q=\frac{\partial z}{\partial y}\right)$

$$1+q = \phi'(x^2+y^2+z^2)\left[2y+2z\cdot\frac{\partial z}{\partial y}\right]$$

$$1+q = \phi'(x^2+y^2+z^2)[2y+2zq] \to (4)$$

$\left(\because q = \frac{\partial z}{\partial y}\right)$

from (3) & (4)

$$\frac{(3)}{(4)} = \frac{1+p}{1+q} = \frac{\cancel{\phi'(x^2+y^2+z^2)}(2x+2z\cdot p)}{\cancel{\phi'(x^2+y^2+z^2)}(2y+2yq)}$$

$$\frac{1+p}{1+q} = \frac{\cancel{2}(x+zp)}{\cancel{2}(y+yq)}$$

$$(1+p)(y+yq) = (1+q)(x+zp)$$

$$y+yq+yp+ypq = x+zp+xq+qzp$$

$$qx+zp+x-y-yp-yq+\cancel{zp}-\cancel{ypq}$$

$$p(z-y)+q(x-z)=y-x$$

$\boxed{Pp+Qq=R}$ which is required 1st order PDE.